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Aggregation in models for natural resource policy analysis

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Aggregation in models for natural resource policy analysis

by

Jay Dee Atwood

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CHAPTER I.

CONSISTENCY IN NATURAL RESOURCE POLICY ANALYSIS

Analysis of current soil and water resource policy most often involves several models (or systems of models) each of which have been built up to differing aggregation levels and behavioral specification. The complexity of the individual models combined with the stringent conditions needed for exact aggregation result in the use of in-exact behavioral correspondences and inconsistent aggregators between the various models. The resource economists involved in the policy analysis exercises are then faced with a real concern over intermodel consistency of results. This study utilizes published aggregation theorems and theoretical model examples to give some model development guidelines for minimizing the consistency problem.

The analytical requirements of current policy analysis are such that a sort of "push-pull" competition exists between the regional partial equilibrium and national partial or full equilibrium models which are commonly used together in the analysis. The resource endowments of a nation are highly regional specific. Policies, either national or regional in origination, also tend to be regionally specific. Hence, the need for regional detail and the capacity constraints associated with trying to include everything in a single national model dictate the use of individual regional models.

However, national analysis is also needed since the consequences of resource policies are national in scope. Production in any or all regions generates externalities, both pecuniary and technological, which move across regional boundaries. For example, sedimentation is readily transported across national boundaries and food chemical residues are consumed nationwide regardless of region of origination. Factor and commodity markets are linked across regions and current government programs have tied resource and commodity policies together. A national spatial equilibrium modeling framework is required to account for these externalities.

Development of one large model which includes both regional detail and national spatial equilibrium characteristics is too costly to be feasible. Developing linked or unlinked regional models and using them as a system is also not feasible since convergence to a consistent national set of solutions is prohibitively expensive if not impossible. These constraints to modeling capacity have resulted in the use of various practical aggregation, decomposition and hierarchical modeling methods which are known to be inconsistent, yet hopefully, good approximations of an ideal model. A critical review of these approximation methods is given as part of this study.

The analytical constraints discussed above have lead to different models being used together for the same policy analysis effort. It is desirable that these models give results as consistent with each other as possible. Partial consistency is achievable by analytically solving for intermodel behavioral correspondences and by the appropriate choice of aggregation methods. Various approximation methods for aggregation have been found, some of which do better in certain situations. Hierarchical and other systems modeling developments have attempted to build on these approximate methods. However, inconsistency has remained an important issue.

As will be reviewed in this paper, consistent aggregation, whether it be in a price index, for a production function or in cost functions, requires homotheticity of the aggregator except for cases of specialized functional forms not easily incorporated or simulated in the type of sector programming models studied here. Though homotheticity is defined later in the paper an example of its implications is given here.

Suppose two models, one containing several regions for each region of the other and suppose the commodity prices for the more aggregated model are taken to be the average marginal production costs across the regions of the less aggregated model. It is also desirable that production and cost in the larger region be the sum of that of the smaller regions. Full consistency of aggregation then requires that each aggregator, P_j which is the price of the jth crop, be a homothetic function with arguments p_{ij} (which are marginal costs of the jth crop by ith region). Homotheticity means that if P_j increases proportionately then each p_{ij} must increase by the same proportion. If each P_j is homothetic then the production aggregators $Q_j = f(q_{ji},...,q_{jn})$ must be homothetic also. From the assumption of homotheticity two undesirable results occur. First, the proportions of each Q_j in national total crop production never change. Secondly, the proportions q_{ij} of the ith regional total production also never change. Clearly, with homotheticity the aggregate model performs the same as a disaggregated one. However, if the assumption of homotheticity were true as shown here then differing regional policy impacts would be non-existent. In the real world differing regional impacts and shifting relative production shares (comparative advantage) do exist. Hence, homotheticity is too strong of an assumption for applied resource policy analysis and cannot be relied on for model consistency.

Since inter-model correspondences and aggregators can never be perfect some guidelines must be developed for the appropriate use of near-consistent methods. These guidelines and implications are developed using theoretical example model comparisons of perfect consistency and contrasting to alternative outcomes of less than perfect methods. These results suggest model development guidelines for policy analysis. These aggregation and behavioral consistency implications are the theme of this study.

The objectives of this study are given below at the end of this chapter. Chapter II gives a synthesis of aggregation theorems, shows how they are impossible and indicates why they are instructive as to the approximate methods one should use. Chapter III outlines some pragmatic methods which have been developed and used and indicates what is missing in them with regard to aggregation and behavioral consistency. Chapter IV presents some theoretical illustrative examples with their "solved for" analytical results giving exact consistency and shows the behavioral implications for natural resource policy models; guidelines for use of these results in defining partial or practical aggregation modeling methods are discussed. Chapter V is a stylized application involving a couple of actual models others have employed in an analytical exercise. This application illustrates the concepts of this study. Finally, Chapter VI gives some conclusions for modeling natural resource policies with regional and national models.

Objectives

1. Organize results on aggregation theory in a form where the implications relative to natural resource models are made accessible.
2. Review and critique some pragmatic efforts which are commonly used to partially overcome the aggregation problem.
3. Construct some theoretical model examples to show implications of assuming aggregation consistency is satisfied and develop guidelines researchers can use to minimize the consistency problem.
4. Take two example models which have been used simultaneously, but not formally linked, in policy analyses and show how the results of this study can be used to improve consistency.

CHAPTER II.

THE PROBLEM OF CONSISTENT AGGREGATION

Nearly all economic analysis involves aggregation. Economists are interested in methods of aggregation which minimize the loss of information and/or error of prediction. Consequently the literature on aggregation has grown in separate but related ways around the entities most frequently aggregated: general functional relationships and their variables (Green, 1964), aggregation of decision variables or steps for a single optimizing agent (Blackorby et al., 1978), aggregation across a set of decision makers (Chambers, 1988), aggregation in sector and/or planning models and, finally, aggregation of heterogeneous performance measures of the economy such as "price" or "output" in the form of index numbers (Afriat, 1981; Diewert, 1981).

This chapter will first define in general terms with an intuitive example what is meant by consistent aggregation along with a brief discussion of the acceptability of less than consistent methods. Aggregation theorems for each of the classes of entities noted above will be summarized and their implications for natural resource modeling shown. Finally, a summary of the chapter is given.

Strict Consistency of Aggregation

With aggregation being necessary it is logical to desire that the aggregated model give the same results as would be obtained from solving the disaggregated model and then aggregating results. An intuitive example of strict aggregation consistency where models with alternative levels of aggregation are being considered can be seen by evaluating the diagram by Ijiri (1971) shown in (2.1).

$$\begin{tikzcd}
{} \arrow[rr, "\text{Optimization}"] & & {} \arrow[dd, "\text{Aggregation}"] \\
X \arrow[r, "\phi(X)"] \arrow[d, "g(X)"'] & V \arrow[d, "\psi(V)"] & \\
W \arrow[r, "h(W)"'] & Y & {}
\end{tikzcd} \tag{2.1}$$

In (2.1) ϕ is a micro function, h is a macro function, g is an active aggregator function and ψ is a passive aggregator function. Consistent aggregation in (2.1) requires that (2.2) hold.

$$(2.2) \qquad \begin{aligned} \phi(X) &= V \Rightarrow h(g(X)) = \Psi(V) \\ \phi(X) &\neq V \Rightarrow h(g(X)) \neq \Psi(V) \end{aligned}$$

An agricultural economics example of (2.1) and (2.2) is if one aggregation and modeling path to national corn output involves first finding the optimal corn production for each farm via ϕ then aggregating the farm optimums via $\boldsymbol{\phi}$ to yield national corn production. The other path uses g(x) to first aggregate the resources and technology of all the farms into a national farm w and then employs h(w) as the optimization process whereby the national farm decides how much corn to produce. Consistency as shown in (2.2) requires that both paths result in the same national corn level. It is also desirable that if policy impacts were estimated, each path give the same result.

Alternatively, consistent aggregation is achieved when the use of more detailed information than that contained in the aggregates makes no difference to the results of the analysis of the problem (Green, 1964, page 3). Green suggests "aggregation will be judged satisfactory by the economist to the extent that he believes that the cost of handling information in greater detail outweighs the greater reliability of the results he might obtain by using more detailed information." Aggregate equations may be more accurate if proper specification of micro relations require inclusion of macro variables (Green). An example is household expenditure functions involving both own and total income.

As will be shown in this chapter to meet the conditions for consistency in a model with a higher degree of aggregation either the data or economic structure must be so homogeneous that either disaggregated results are directly derivable from an aggregated model or the aggregates are of a form with little economic meaning. Comparison of strict aggregation methods to alternatives in example models in the following chapters enables development of guidelines for empirical work.

Empirical models implicitly involve aggregation of prices, inputs, technology and firms in both a temporal and spatial sense. This chapter deals with these concepts in a more general model framework than the restricted (from a rigorous theoretical view but detailed by other standards) technology of a typical sector programming model. However, standard micro-economic entities such as factor supplies and substitutability, product demand and product supply curves are implicit in LP sector models, whether explicitly built in or apparently derivable from parameterization techniques. Therefore, it could be implied that the simple analytical models used in the following chapters are really simulating aggregate outcomes of the programming models. The aggregation concepts discussed below seem generally to have been assumed unimportant in most programming model studies. The results of this study should bring more attention to the modeling aggregation issue.

Functional Relationships/No Economic Restrictions

The strictest conditions for aggregation consistency arise when independent variables are free to take any values and consistency relies solely on functional form and/or correlation between variable movements over time (Green, 1964, Chapters 5 and 6). If both dependent and independent variables are assumed to be simple sums, the individual functions being aggregated must be linear with identical slopes. If either the dependent or the independent variable is allowed to be a weighted sum, the functions must still be linear and the ratio of slope coefficient to weight must be constant across all functions. A consumer example is the requirement that the ratio of income to marginal propensity to consume (the weight) be constant across all households for the aggregate consumption function to be consistent. Production functions must be linear and homogeneous with intercept terms equal to zero and exhibit parallel hyperplane isoquants, identical across all firms.

If log forms can be taken and used in place of non-linear functions then the log functions can be treated as linear functions for aggregation. Log functions themselves restrict variable behavior and sums of logs of variables rather than sums of variables are more difficult to work with. For arbitrary second degree polynomials complex weighting restrictions are essentially that all own variable second degree term coefficients and all cross term coefficients be equal across all functions (Green). Weighted moments of first and second order can be used in some econometric work (Green; Theil, 1954).

For a Single Decision Maker Given Economic Restrictions

When some conditions on movements of variables are found from micro-economic theory for optimization by a single decision maker and imposed, it is the case that the strictness of the aggregation conditions are only a little relaxed. These behavioral restrictions generally involve such untenable restrictions on technology or allowable structure as to make the model less useful.

The first concept considered is weak separability. Weak separability for a subgroup of variables requires that the ratio of first derivatives of the function for any two variables in the subgroup be independent of movements of any variables not in the subgroup. Changes in the variable out of the subgroup can shift the isoquant of the two variables but cannot change its slope (Chambers, 1988). This is shown in (2.3).

$$(2.3)\quad \frac{\partial}{\partial x_{qi}}\left(\frac{\partial f/\partial x_{rj}}{\partial f/\partial x_{rk}}\right) = 0 \text{ for all } q,r,i,j,k \ (q,r = 1,\ldots,m: \quad q = r;\ i = 1,\ldots,n_q;\ j,k = 1,\ldots,n_r).$$

Strong separability requires that the ratio of first derivatives of any two variables be independent of movements of any other variables (whether in the same subgroup or not). These separability concepts are most commonly referred to with regard to utility, production, cost and profit functions.

An intuitive example of separability actually existing arises from the observation of two-stage optimization. In the consumer case, elementary goods are first assigned to groups. Total expenditure is allocated to these groups in the first stage of optimization. The second stage involves allocating each group expenditure among commodities in that group, independently of other group allocations. Similar examples exist for multi-stage production processes. As an illustration, functional separability allows (2.4) to be written as (2.5).

(2.4) $y = f(x_{11},\ldots,x_{1n},\ldots,x_{ij},\ldots,x_{mn_m})$

(2.5) $y = F(X_1,\ldots,X_i,\ldots,X_m)$ where for each r $(r = 1,\ldots,m)$

$$X_r = f_r(x_{r1},\ldots,x_{rk},\ldots,x_{rn_r})$$

Overall consistency of the two stage decision process requires conditions on both the variable groupings in the second stage and on the first stage decision function and its group aggregates. The desired conditions are that the two stage process give the same result as optimizing (2.4) directly and that each group price index multiplied by the corresponding group quantity index give group expenditure.

Separate conditions are required for the aggregation of variables in the second stage of the decision process and for the overall two stage result to be consistent (Blackorby et al., 1978). For optimization within each of the second stage groupings to be consistent weak separability of (2.4) for the groups as shown in (2.5) is necessary and sufficient. If only two groups are involved weak separability is also sufficient for the overall two stage process.

If more that two groups are involved then stronger conditions are needed for consistency of the overall two stage process. Either the macro function F in (2.5) must be strongly separable (additive) in the group quantity indices or F must be weakly separable in the indices and each indice be homothetic (defined in next paragraph), i.e., F must be homothetically separable.

A homothetic function is defined as a transform of a linear homogeneous function (Chambers, 1988). If the transform goes to infinity when the original goes to infinity and if the value of the transform evaluated at zero is zero then the transform will have the same basic properties as the original. There are two important properties of homothetic functions which are relevant to aggregation theory. First, proportional changes in all inputs are accurately reflected by the same proportional change in the aggregate. Second, the marginal rate of substitution between two inputs is homogeneous of degree zero since it is the ratio of the first derivatives of a function homogeneous of degree one. This implies that expansion paths are straight lines through the origin. Later in this chapter quasi-homothetic functions are mentioned. In this case the expansion path is a straight line, but not through the origin.

The set of functional forms satisfying these restrictions is small. Also, data are not generally collected, summarized and made available in forms satisfying these restrictions. As an example, the Cobb-Douglas function is homothetic. However, in this context for the Cobb-Douglas the quantity indices are weighted geometric means of the group commodities with the weights being the constant expenditure proportions. The price indices are also weighted geometric means. Aggregate data that are available are generally sums or weighted sums.

The group expenditure requirement is also met if prices and quantities of a group always move proportionately (Green). Where group separability is based on price proportionality, there is no reason to believe that in the absence of it that the group would even be weakly separable; the equal product surfaces or indifference curves implied by these conditions will in general be different for different values of variables in other groups.

With the restrictions on technology given here it is as if each X_r in (2.5) is also a production function, satisfying the same properties as y, i.e., homogeneity (Chambers, 1988). For production functions the marginal rate of technical substitution between any two inputs in the weakly separable group must be independent of utilization of an input outside the group. When technology is strongly separable (2.5) may be written as (2.6) and it can be seen that the aggregate inputs must be perfectly substitutable for one another in production.

$$(2.6) \quad f(x) = F^{*}\left(\sum f^{i}(x^{i})\right)$$

The literature on cost, profit, and production functions expand and refine the two-stage budgeting, separability, and strict aggregation concepts defined above for various applied and empirical situations (Blackorby, Primont, and Russell, 1978; Shephard, 1970). Cost and profit functions are most frequently used in econometric work but there are some duality results applicable to this programming model study which can be shown with a cost function.

If F, f, f_i, and x are replaced with C, f, c_i, and w making (2.4) and (2.5) a cost function then weak and strong separability of inputs and prices are as defined above. C and each c_i are called the "macro-function" and "aggregator functions". If C is also separable in output then (2.7) holds.

$$(2.7)\quad \text{If } \frac{\partial}{\partial y}\left(\frac{\partial c/\partial c_i}{\partial c/\partial c_j}\right) = 0, \text{ then } C(w,y) = c(y,c^*(w)) \text{ and}$$
$$C(w,y) = c(y,c_1(w_1),\ldots,c_m(w_m))$$

The implication of (2.7) is that each $c_i(w_i)$ is simply a price index, though satisfying properties of and being a cost function.

Weak and strong separability imply that if f has certain properties, its macro and aggregator functions can be specified so as to possess those properties. Duality theory shows that if the cost or profit functions possess certain properties, then statements about the production technology properties can be made. Duality theory allows testing for separability in production processes via estimation of cost or profit functions followed by testing of price separability (a nice example is Fuss, 1977). If separability exists and aggregates can be formed, they will not be simple averages or summations but will depend on the form of the production or cost functions they are used in (Berndt and Christensen, 1973; Shephard, 1970; Blackorby, Primont and Russell, 1978; Chambers, 1988). Empirical cost and profit function estimations reported in the literature for U.S. agriculture can provide useful guidelines for constructing acceptable aggregation structures in resource policy models.

The conclusion of this section is that the economic restrictions arising from satisfaction of the first order conditions (FOC) of optimization can allow consistent aggregation; however, they impose restrictions on allowable economic behavior and/or technology. Separability facilitates aggregation but only a restricted set of functional forms are consistent with separability.

Across Individual Decision Makers

The results presented here are from Chambers (1988, pages 182-199). First, Chambers' general aggregation results are presented, followed by his cost function examples. This brief summary of Chambers' results provide rules which the following chapters of this study refer to.

The problem is to find an aggregate function $g(z,k)$ (cost, production, or profit), where z are the variables to be aggregated and k is an arbitrary constant, such that (2.8) holds.

(2.8) $g(z,k) = H(g^1(z^1,k),\ldots,g^m(z^m,k))$ where $z = z(z^1,\ldots,z^m)$

Different results are given depending on whether the aggregation rule, H, is linear or non-linear and for the case of linear H whether z is linear or not. For the cost function example these are discussed case by case.

Linear H and z

Linear H and z means that aggregates are constructed as $H = g^1 + g^2 + \ldots + g^m$ and $z = z^1 + z^2 + \ldots + z^m$. In this case the aggregate function must be of the form $g(z,k) = v(k)z + m(k)$ where each $g^i = v(k)z^i + m^i(k)$ and $m(k) = m^1(k) + \ldots + m^m(k)$.

For cost functions these linear aggregation rules are shown in (2.9). These linear aggregation rules for cost functions seem desirable, i.e., industry costs equal the sum of firm costs while industry output equals the sum of firm output, however, they impose strict conditions on allowable technology. It is also implied that the actual distribution of output across firms does not matter.

(2.9) $c(w,y) = c^1(w,y^1) + \ldots + c^m(w,y^m)$ for $y = y^1 + \ldots + y^m$.

With these linear restrictions it is found that: (i) each firm level marginal cost (MC) must equal aggregate MC; (ii) since (i) must hold for any level of y^i, each firm level MC must be independent of y^i since moving output from one firm to another must not affect costs; (iii) it is implied that MC for firm i and j must be the same; and, (iv) aggregate MC must be independent of aggregate output.

To get these linear results the aggregate cost function must be of the form $c(w,y) \doteq \lambda(w)y + c^*(w)$ where $\lambda(w)$ is the MC and $c^*(w)$ is a constant of integration (this is the Gorman Polar form (Gorman, 1959; Deaton and Muellbauer, 1980)). Such a cost function requires quasi-homothetic technology (i.e., a linear transformation of a homothetic technology). The cost function must be affine in output and for it to satisfy the properties normally postulated for cost functions (particularly $c(w,0) = 0$) requires $c^*(w) = 0$ which then gives actual linear homogeneity of production. One could assume $c(w,y)$ is a long run cost function ($c^*(w)$ are fixed costs which equal zero in the long run) but then normal U shaped average cost curves cannot be represented. Note that homothetic technology also requires $c^{*}(w) = 0$.

The above results on aggregate forms also imply restrictions on allowable firm level functions. Technical differences between firms are restricted to the c^{i*} terms; however, again to satisfy cost function assumptions the c^{i*} terms must be equal to zero. The conclusion is that it is not possible to specify firm level technologies satisfying conditions of cost functions that allow for consistent aggregation across firms with non-identical technologies in the long run. Only identical constant returns to scale (CRS) technology satisfy linear aggregation in the long run. Any consistent short run cost functions violate normal conditions on cost functions since they include the $c^{*}(w)$ terms.

A graphical interpretation of these conditions was given much earlier in terms of Engel curves or expansion paths in the following two theorems (Green, 1964, pages 47-50):

> "Theorem 9. It is necessary for consistent aggregation, when the optimal conditions (6.1) are satisfied, that: (a) for each firm or individual, each set of points, in output or commodity space, at which marginal rates of substitution are constant, is a straight line; (b) for a given set of marginal rates of substitution, the straight lines for all individuals or firms are parallel."
>
> "Theorem 10. If the Engel curves or expansion paths for all individuals or firms at a given set of commodity or input prices, are parallel straight lines through their origins, then consistent aggregation of the functions $f_s(x_{1s},\ldots,x_{ms})$ to the functions $y = F(X_1,\ldots,X_M)$ is possible. Moreover, there exist functions F and $h_1,\ldots,h_m$ such that
>
> $Y = \sum h_s(y_s) = F(X_1,\ldots,X_m)$

> where the function F is homogeneous of degree 1 in its arguments."
>
> "If the conditions of Theorem 10 are satisfied, and each of the functions f is homogeneous of degree one, consistent aggregation is possible with
>
> $Y = \sum C_s Y_s$

Linear H and non-linear z

If one is willing to use $y = a^1y^1 + \ldots a^my^m$ as a weighted total production the marginal costs across firms no longer need be the same. However, each firm level MC must be constant and equal to aggregate marginal cost times the appropriate weighting factor; aggregate marginal cost must still be constant and independent of output. So even here the conditions on allowable technology are still strict.

Non-linear H

Allowing for non-linear aggregation rules imply the following conditions: (1) weak (strong) separability of z imply, and is implied by, weak (strong) separability of H in g^i; (2) homotheticity of z implies, and is implied by, homotheticity of H in z; and (3) H is additively separable ($H_{ij} = 0$).

A non-linear aggregation rule for cost functions is given in (2.10). In this case y could be thought of as representative output level rather than actual aggregate output.

(2.10) $c(w,y) = c^1(w^1,y) + \ldots + c^m(w^m,y)$ where

$c^i(w,y^i) = h^i(y^i)\lambda(w) + c^{*i}(w)$ and $y = h^1(y^1) + \ldots + h^m(y^m)$.

Now aggregate MC need not be independent of aggregate output but aggregate cost must be additively separable in firm level outputs. If (2.10) is true then aggregate MC need not be independent of aggregate output but aggregate cost must be additively separable in firm level outputs. If (2.11) holds then aggregate MC is independent of y.

(2.11) $\partial^2 y / \partial y^i \partial y^j = 0$

Here, each firm level cost function is homothetic but MC is not identical across all firms and is not independent of firm level output. If (2.11') holds instead of (2.11) then so do (2.12) and (2.13).

(2.11') $\partial^2 y / \partial y^i \partial y^j = 0$

(2.12) $c(w,y) = h(y)c(w) + c*(w)$ where $h(y) = h^1(y^1) + \ldots + h^m(y^m)$

(2.13) $c^i(w,y^i) = h^i(y^i)c(w) + c^{*i}(w)$

In this case: (1) MC is not constant either for the aggregate or for individual firms; (2) homothetic production structures can be represented without imposing CRS; (3) can now aggregate different technology across firms because both $h^i(y^i)$ and $c^{*i}(w)$ can vary; (4) even if $c^{*i}(w)$ is forced to zero to be consistent with cost function properties, technology can vary across firms so long as the $h^i(y^i)$ terms are different.

The conclusion is that consistent non-linear aggregation in the longrun requires that each firm-level production function is a transform $(F)^i$ of the same linearly homogeneous function $f^*(x)$. So, input requirement sets are still required to be parallel across firms. For aggregation across decision makers, one must allow the aggregator to not be simple sums of entities (i.e., national corn production not the sum of regional corn production) to get even close to consistency.

Linear Programming Model Aggregation Rules

A most ideal sector programming model would include full detail on each producer in the economy. In practicality analysis proceeds with either first aggregating the decision components (resources, technology, and net returns expectations) of producers into groups and solving a model with the group aggregates or independently solving each producer's decision problem and then aggregating results. For our discussion the decision components considered in an linear programming (LP) model aggregation framework are defined in a profit maximization model for the ith producer in (2.14) with the dual given in (2.15).

$$\text{(2.14)}\quad \underset{X_i}{\text{Maximize}}\ Z_i = C_i'X_i$$

$$\text{subject to:}\quad A_iX_i \leq b_i$$

$$X_i \geq 0$$

where:

$i = 1,\ldots,I$ for the firms to be aggregated,
Z_i is net optimal returns for the ith firm,
C_i is vector of activity net returns for ith firm,
A_i is m*n technology matrix of ith firm,
X_i is vector of activity levels, and
b_i is m*1 vector of constraints (resources etc.).

$$\text{(2.15)}\quad \underset{R_i}{\text{Minimize}}\ V_i = R_i'b_i$$

$$\text{subject to:}\quad A_i'R_i \geq C_i$$

$$R_i \geq 0$$

where:

R_i is m*1 vector of resource shadow prices,
and other items are as defined in (14).

A comprehensive set of strict aggregation conditions were proposed by Day (1963, 1969a, 1969b):

(a) $A_i = A$, i.e., technological homogeneity,

(b) $C_i = C$, i.e., pecunious proportionality of net returns expectations,

(c) $b\ = \lambda_i b$, $\lambda_i \geq 0$, institutional proportionality.

Institutional proportionality is strictly only necessary for the constraints binding in the model solution, but difficulty exists in identifying them beforehand. Since Day's conditions ensure $X_i = \lambda_i X^*$ for all farms they are usually sufficient. Strict sufficiency requires $\sum \lambda_i = 1$, i.e., representative or average farm is the arithmetic mean (Hazell and Norton, 1986) and that none of the individual farm models be degenerate since then

X^*_i is unknown and arbitrary over a range (Spreen and Takayama, 1980).

Less stringent conditions for aggregation are proposed by Sheehy and McAlexander (1965), Miller (1966), and Lee (1966). All of these rely on all farms included in a group having coefficients within the tolerant range of the basis of the group farm model (Hazell and Norton, 1986). Since the tolerant ranges change for alternative experiments these methods are impractical in policy analysis. The conditions of Paris and Rausser (1973) also require knowledge of the solution space prior to grouping. The results of Marenco (1969) and Guccione and Oguchi (1977) also aid policy analysis very little. The best approach seems to be to use empirical data to determine factors for determining aggregation characteristics which are most likely to remain constant across scenarios to be evaluated.

Index Numbers as Summaries of Detailed Information

Index numbers are often used in economic analysis as aggregate measures to summarize more detailed information. The most common indices appearing in natural resource policy models are for commodity prices and yields. The indices can be constructed to represent aggregation across regions, time periods, producer classes, and other desired economic variables. The major problem in developing these indices for a policy analysis model (as explained in more detail below) is in choosing a formula and its coefficients before the policy impacts are known. Hence, a major part of the research on index numbers has been to search for formulas whose coefficients (weights or shares) are independent of the policy scenario. However, as will be shown, the sufficiency conditions for independence impose such strict conditions on the form of the underlying behavior that the policy model is unuseful.

This entire index number section is based on a survey article by Diewert (1981). Diewert's results are interpreted in terms of their value for natural resource policy programming models. As an example of indices and models, suppose all producers in Iowa are aggregated and their collective behavior simulated by a profit maximization programming model. An Iowa corn price (or yield) is really non-existent. What really exists are prices and yields by region, quality, season, etc. A common practice is to form the state price and yields as some average, an index, of the component types. However, the question is then whether the averaging method be based on past sub-state regional distributions or on those predicted for the future? The problem is in specifying the distribution for model development before the policy impacts for which the model is being developed are known.

The most straightforward test for consistency in indices (but which is also very difficult to meet and the value of which has been questioned by some) is Fisher's weak factor reversal test (Fisher, 1922). The weak factor reversal test for indices between two scenarios is shown here in an example to illustrate the notation used in the following discussion:

$$P(p^o,p^1,x^o,x^1) * Q(p^o,p^1,p^o,p^1) = p^{1T}x^1/p^{oT}x^o$$

where $p^{iT}x^i = p_1{}^i x_1{}^i + p_2{}^i x_2{}^i + \ldots + p_n{}^i x_n{}^i$

p and x without subscripts are vectors and a T superscript indicates the transpose

p^{ij} = price of ith good in jth category

x^{ij} = quantity of ith good in jth category

Various indices have been used in economic analysis. The Laspeyres indices use base period commodities (or prices) as weights while the Paasche use current period entities. The Marshall and Edgeworth, Fisher, Tornqvist and Divisia indices all attempt a sort of endogenous weight (or weighted average of two periods) (Shepherd, 1963). All of these indices are inconsistent for many of the uses index numbers are applied to. In particular, these indices are limited for estimating the change in the cost of a given level of utility or production. In other words whether or not consumers are better off in the new or the old scenario or whether the production level is higher or lower cannot be answered unless homotheticity of the utility or production function is present. Unfortunately, introducing a utility or production level interpretation to the index number problem creates most of the problem of finding consistency. As will be shown, the next step after introduction of output or utility into the indices is to look for functional forms such that utility or production level drops out of the index formula. Functional forms giving that condition are shown to be too restrictive for economic analysis.

Assume that the aggregation is being done for a decision maker who chooses a value for x to satisfy the problem of maximizing the value of an "aggregator" function, $u = f(x)$, subject to a budget constraint in each scenario (f can be utility or production level). Assume that f has the properties of: (i) continuity, (ii) increasingness, and (iii) quasi-concavity. Under some cases it is also assumed (or required for some results) that f is also "neo-classical", i.e., f is positive, positively linearly homogeneous, and concave.

Assume that in the optimization process, first for any level of output, u, cost is minimized such that a cost function, $c(u,p)$, can be defined. Secondly, given the cost function, the level of $f(x)$ that will maximize u is determined. It is assumed that $c(u,p)$ satisfies the standard cost function properties given by Diewert (1981, page 164). Note that if f is neoclassical then $c(u,p) = u * c(1,p) = u * c(p)$.

With these definitions a true price (cost) index can be defined for a reference vector x as: $P(p^o,p^1,x) = c[f(x),p^1]/c[f(x),p^o]$. The problem is then in choosing a functional form of f such that $P(p^o,p^1,x) = c(p^1)/c(p^o)$, i.e., a functional form such that the reference vector chosen for the index is unimportant. If such a functional form cannot be found, invariably error will be introduced into policy analysis as inexact indices must be used (the most famous examples are the biases of the Paasche and Laspeyres indices when they are used to estimate cost of living changes). It is required that f(x) be homothetic for x to drop out of $P(p^o,p^1,x)$. For the case of a non-homothetic f various reference vector and index formula propositions have been made. Diewert summarizes these propositions and shows how only bounds can be determined for the exactness of the results. With these formula and their bounds the sort of desirable properties examined by Diewert are as an example, if all prices increase, then the index must also increase, and so on.

The real contribution of Diewert's work is to link aggregator functional forms with exact and superlative index formulas (superlative index formulas are those which are exact for functional forms that are capable of providing second order approximations to arbitrary aggregators). A quantity index is defined to be exact if $Q(p^o,p^1,x^o,x^1) = f(x^1)/f(x^o)$ and a price index is exact if $P(p^o,p^1,x^o,x^1) = c(p^1)/c(p^o)$. Note that as a general rule if $f(x)$ is homothetic then P and Q are exact. Some specific examples of functional forms meeting these restrictions are given below.

The Paasche and Laspeyres price indices, $P_p(p^o,p^1,x^o,x^1) = p^{1T}x^1/p^{oT}x^1$ and $P_L(p^o,p^1,x^o,x^1) = p^{1T}x^o/p^{oT}x^o$ and the Paasche and Laspeyres quantity indices, $Q_p(p^o,p^1,x^o,x^1) = p^{1T}x^1/p^{1T}x^o$ and $Q_L(p^o,p^1,x^o,x^1) = p^{oT}x^1/p^{ot}x^o$ are exact for the Leontief aggregator function $f(x) = \min_x\{x_i/b_i:i=1,\ldots,n\}$ and the linear aggregator function $f(x) = a^Tx$. Therefore, in the type of price and yield examples given thus far in this paper, if the weights assigned to each component entity are not expected to change regardless of the scenario considered the Paasche and Laspeyres indices are exact and aggregation will be consistent. However, if production patterns were actually fixed such that constant proportions (weights) prevail the resulting policy analysis would yield little useful information. Note that according to Diewert's results the Paasche and Laspeyres indices normally form the outer bounds for other indices, i.e., there are modified indices available which are guaranteed to do no worse than the Paasche or Laspeyres.

A family of geometric price and quantity indices can also be defined using shares as the power coefficients. These indices are shown by Diewert to be exact for f being of the Cobb-Douglas form. However, for the purpose of defining regional prices and yields in a regional model as averages of sub-regional data, geometric price and quantity indices are confusing (i.e., they cannot be compared to published regional statistics and model interpretations are difficult).

The limitations of the Paasche (present period weights) and Laspeyres (last period weights) for showing changes in costs associated with holding the aggregator at a constant level are well known. The Fisher Ideal index, $P_f(p^o, p^1, x^o, x^1) = (p^{1T} x^1 / p^{oT} x1)^{1/2} * (p^{1T} x^o / p^{oT} x^o)^{1/2}$ and similarly for a quantity index are attempts to overcome these weaknesses. These indices are exact if f is homogeneous quadratic of the form $f(x) = (x^T A x)^{1/2}$ where A is a matrix of constants.

A quantity index is superlative if it is exact for an aggregator which is a second order differential approximation to an arbitrary twice continuously differentiable linearly homogeneous aggregator function (i.e., for "flexible form" aggregators). A price index gets the same label if it is exact for a unit cost function which likewise provides an approximation to an arbitrary twice continuously differentiable cost function. The Tornqvist indices (in various forms) are exact for general translog functional forms which may be non-homothetic if certain other restrictive properties hold. However, translog functional forms have not been employed in programming models and would be difficult to both implement and interpret.

In conclusion, aggregators suitable for developing coefficients in aggregate level resource policy models will most likely be biased. This is because a Leontief or linear aggregator will be used and it is unlikely that the underlying economic processes are truly homothetic. Disaggregation of the aggregate measures to more a more detailed level requires very strong assumptions not likely supported by empirical observation. The best guideline then is to make use of empirical information in choosing aggregator coefficients so as to minimize error given the aggregator forms used. An example of this is the "aggregate by most limiting resource" rule of linear programming models or the value weighted rule for composite commodities (Hazell and Norton, 1986).

The Importance of Homotheticity

In conclusion for this chapter, homotheticity plays a major role in defining exact consistency in aggregation. Consistent aggregation requires restrictions on technology or preferences which are essentially equivalent to homotheticity; homotheticity is then often sufficient for consistent aggregation. Non-homothetic aggregators which are consistent are available; however, they result in aggregates with interpretations differing from the "sum" or "total" or "average" needed in resource policy models at the aggregate level.

The next chapter reviews the modeling literature to illustrate pragmatic attempts to bypass these stringent aggregation conditions. The fourth chapter presents some pragmatic analytical models simple enough to work with where guidelines for minimizing the problem of consistency can be derived. With those models the implications of assuming consistent aggregators and proceeding with policy analysis can be shown. Also the implications of proceeding with inconsistent aggregators can be compared.

CHAPTER III.

APPROXIMATIONS TO IDEAL SECTOR POLICY PROGRAMMING MODELS

The previous chapters hinted at but did not make it clear that the focus of this study is on programming rather than on other types of models. Econometric models are referred to only as needed such as in cases where they supply parameters for the programming models. Mathematical programming models (MPs) are often favored for natural resource policy analysis over econometric models because of several reasons. MPs may be specified to explicitly account for the optimization behavior of individual producing, consuming and policy making units at whatever degree of detail that is desired. MPs allow inclusion of vast resource data detail: spatial, temporal, producer class, resource quality, etc. MPs can also be set up to include the desired degree of rigor with regard to micro economic theory; however, there is generally a required tradeoff between theoretical rigor and detail in other areas. Most importantly, through the use of engineering cost and other data, MPs allow inclusion of new options for producers, consumers and policy makers which have not been seen historically and for which data do not exist.

For the type of resource policy analysis addressed here there are several distinct disadvantages to using econometric models (Norton and Schiefer, 1980a): in multi-product and/or multi-regional situations there are insufficient degrees of freedom to estimate all desired parameters; the econometric estimates are only valid over a range of historically experienced variation which usually differs from the policy environment under question; the models cannot account for inequality constraints such as for excess land; and they do not have the capacity to provide much complementary information on movement of other variables of interest in the problem.

There are also weaknesses in the use of programming models, some of which are addressed in this study (Shumway and Chang, 1977). The data and manpower requirements and computer capacity are generally large for national or regional models compared to less detailed (more aggregated) econometric models. With linear models the curse of dimensionality is encountered when non-linear relationships are approximated or when combinatorial types of options exist (such as where 16 types of tillage for 10 rotations on 24 landgroups in 105 regions are modeled for a nation). With non-linear models current computational capacity limits the scope to relatively few variables and rather simple relationships. MPs rigorously honor the "marginality" conditions of optimization and since models are abstractions of reality and include only a subset of the constraints decision makers actually face they will generally give a more "efficient" solution than can the economy.

The natural resource policy models which this study evolves around are primarily used to evaluate policies and options not seen in the past. Policies are represented in an MP by new activities, new constraints, right-hand side changes or factor supply and product demand modification (McCarl and Spreen, 1980). For example, new activities can represent the new technology arising from sponsored research or extension; economic development projects can augment available resources; quotas, subsidies, tarriffs, etc., can affect supply and demand; and laws can take the form of constraints.

With regard to new constraints representing policy provisions several authors have warned that the distinction between constraints as actual limitations seen by the individual decision makers and as policy goal targets must be carefully considered (Hazell and Norton, 1986; McCarl and Spreen, 1980). The solution will rigorously honor the constraints hence their use must be restricted to actual impositions on individual behavior. Otherwise, the model meets the constrained policy goal as if one national decision maker were in place and the analyst is left with no knowledge of what the producers and consumers would actually have done with regard to the goal.

Many of the historical MP model specifications really look like one giant farm model and simulate the entire agricultural sector as if it were under the control of one decision maker.

Most of the cost minimization applications have been made as if the agricultural sector were in a long run competitive equilibrium which implies profit maximizers act as if they are minimizing the average cost of production. Assuming such a scenario implies the models are inappropriate for policy analysis where profit incentives are involved. The assumption of one decision maker has also often been implicitly invoked in these long run cost minimization and in other classes of models with the imposition of policy goals in the form of constraints (Hazell and Norton, 1986).

Ideally, the MP models should have represented the aggregative effect of all the individual producers and consumers in the economy. In actuality, constraints imposed by data availability, computational capacity and research budgets have resulted in more simplistic models.

A brief review of how economic analytical tools are formed and where weakness occurs shows in general where a structured hierarchical system of programming models could make a contribution to more ideal sector policy analysis. In the first step, a complete conceptual model of how the economy works and how policy intervention impacts occur is formulated. Second, an economic analytical tool or model is chosen to approximate the conceptual model. Third, the constraints imposed by data availability, computational capacity, and research budget result in the chosen analytical tool being simplified. The biggest simplification in programming models is usually because increases in model detail must come at the expense of capacity to determine simultaneously both prices and quantities. Current approaches of overcoming these constraints involve independently addressing segments of the overall resource policy problem with unlinked models of very different nature.

Large detailed linear programs (LP), respecified for each different set of prices and generally lacking in input and product substitutability, non-linear programs (NLP) which can determine both prices and quantities but have more limited capacity for disaggregation, and linear simulated non-linear formulations with LPs (also with limited capacity for disaggregation) have been used (Takayama and Judge, 1971; Heady and Srivastava, 1975; Meister et al., 1978; Norton and Solis, 1983; and Hazell and Norton, 1986). These approaches often result in inconsistent estimates of impacts on prices, quantities and welfare measurements. In some cases the final analytical tool gives results that are even qualitatively different than would be predicted by the conceptual model. In any case validation of these models is a difficult problem in itself leading to question of credibility of the results (Nugent, 1970; McCarl and Apland, 1986; and Hazell and Norton, 1986).

Linear programming models can be structured to capture nearly any economic behavior given computer capacity and technical skill (or if detail in some other area can be traded off). Examples are cost minimization, profit maximization, surplus maximization (or market rent minimization), general equilibrium and other special applications. Non-linear programming (NLP) models capture some aspects of the policy problem more efficiently than do LPs but a lot of detail must be given up in other areas.

All of the models reviewed below represent some sort of aggregation process and include a set of behavioral assumptions. The question needing answered is how far these approximations deviate from reality. Note that there are two aspects to this problem. First, the model may have some theoretical weaknesses which result in inaccurate estimates. Secondly, the model may have a correct specification but be supported by weak empirical data. The general focus of this review is on the theoretical aspects since empirical data and resources are lacking for empirical comparisons and tests.

Excellent reviews of the use of programming models for agricultural policy analysis are given in Norton and Schiefer (1980a and 1980b), McCarl and Spreen (1980) and Hazell and Norton (1986). In particular McCarl and Spreen focus on the microeconomic foundations, the embodied theoretical aggregation processes and interpretation of the objective function for sector programming models.

The outline of this chapter is as follows. First, a section on ideal sector programming models versus reality is presented to illustrate the need for the development of the type of concepts addressed in this study. Secondly, individual descriptions of various methods of approximation to the ideal sector model will be given. These descriptions will be accompanied by a critique in terms of the consistency conditions presented in earlier chapters. Finally, a conclusions section is given.

Ideal Sector Models as Contrasted to Reality

Even though programming models have been used in agricultural policy analysis for more than 30 years, better approximations of the sector's economic activity are still searched for. McCarl and Spreen (1980) outline the theoretical reasons for use of aggregate programming models in sector policy analysis. Particularly relevant to this study is their definition of a "price endogenous" programming model (page 90):

> "We may now state the underlying premise for the aggregate model. The production level of each activity should be determined by the first order conditions with which an individual producer will select his production level. Additionally, demand and supply relations should be included. This leads to an aggregate model wherein participants individually behave as small competitive units, yet collectively, price and quantity are endogenous."

A fully ideal agricultural sector policy model includes the following five items (Hazell and Norton, 1986, page 136-137):

1. A description of producer's economic behavior, i.e., their decision rules.

2. A description of the technology or production functions available to the producers.

3. A definition of the resources controlled by the producers.

4. A specification of the market environment the producers exist in, and

5. A specification of the policy environment of the sector.

Obviously tradeoffs exist between these five components, i.e., more detail on the available technology limits the detail allowable in the market specification. How to best address each of these five points for any particular policy analysis depends on available model development and application budget, availability of data and other resources, and on the questions needing answered.

The specific model reviews given later in this chapter show that in general there are tradeoffs between the five components and that inclusion of all in one model may be infeasible. Generally economists have relied on the technique of specifying a single large model for a particular analytical task. With this approach analytical capacity has been a limit to the allowable scope and detail of the model. Contrary efforts have included either decomposition of the large model into several small ones or use of several non-formally linked models. For either of these methods to work such strict aggregation consistency conditions are needed that the analysis is nearly useless. A hierarchical system could be an improvement over these past approaches.

Often good models are developed for a specific or general purpose and then applied to whatever policy question that happens to come along via ad hoc structural changes. As noted earlier, care must be taken in specifying the constraint set of the model to separate out policy goals from the real constraints facing decision makers. With policy constraints included, the model will give a technologically feasible solution but no indication of the type and level of economic incentives required to get voluntary achievement of the goals will be available. The policies must be incorporated into the model in such a manner that the model criterion function reacts the same as do real world producers.

The real constraints faced by economists have resulted in three weaknesses with the existing modeling frameworks and applications. First, the current models are weakly specified as to the decision makers and what variables they can control. As an example, there are really at least three different groups of decision makers involved in soil and water policy. The National Conservation Program (NCP), the Agricultural Conservation Program

(ACP) and other policies are administered by national administrators who lobby for a national budget, determine national priorities and allocate funds to state offices. State administrators, with recommendations from local political groups, plan state programs given the budget allocations and priorities coming from the national office. Finally, the producers, who are ultimately responsible for implementing conservation measures, are seeking to maximize profits rather than minimize environmental degradation. The economic environment in which decisions are taken, the variables to be maximized (or minimized) and the constraints faced are different for each set of actors. The current model specifications where it is implicitly assumed that one decision maker has total control can indicate a physically and biologically feasible outcome. However, it rarely gives an indication of the economic measures required to achieve the desired national planner's goals given the diverse underlying decision functions described here.

Candler et al. (1981) note that the programming for policy analysis literature has in general failed to recognize that when all variables are not under the control of one decision maker large errors may occur by assuming a central decision maker. This criticism applies to most of the models reviewed in this chapter.

In addition, policy makers have become more sophisticated, searching for optimal policies rather than choosing between feasible options. Using current models and evaluating the several feasible alternatives gives no indication whether any one alternative is close to an optimal policy or not. For finding optimal policies when the choice variables and objective functions of the policy makers are distinctly different from those of the economic decision makers a two-level modeling effort is required (Candler et al., 1981). Except in specialized situations the two-level models are difficult to apply and usually only limited trade-off curves between the goals of the policy makers and of the decision makers are possible (Candler et al., 1981; Norton and Schiefer, 1980a).

Secondly, current national soil and water policy models are very large. They contain much detail in the form of regions, resource types, etc., and there is little capacity left for modeling various economic decision behavior regardless of which institutional or producer behavior is considered. Since the economics incorporated in these models is rather limited their major functions are more as accumulators of information or accounting devices. They determine detailed resource allocation and production outcomes given some pre-specified price or quantity variables. The problem with this approach is that the detailed outcomes of the model are not necessarily consistent with the pre-specified information. For example, if demand is pre-specified in the model as quantity and a solution found, the imputed prices coming out of the solution do not match the specified quantities in the original demand system. The overall result is that these models generate little economic information that was not already known in the model development phase. Ideally, the models should include both producers and consumers acting as if they are competitive price takers and endogenous market prices dependent on the aggregate decisions of the individual producers and consumers.

Third, since the Food Security Act of 1985 came into being, conservation and farm income and price support programs have become more intertwined (Glaser, 1986). This has resulted in an increase of financial incentives for producers to implement conservation. The financial incentives for conservation depend to some extent on the aggregate and individual levels of conservation already applied and the national commodity demand-supply balances. The administrators of the commodity price programs (another set of decision makers), are also now more heavily involved in targeting conservation decisions. Attention is now focused on the need to reflect producer response in evaluating conservation and commodity policies which are intertwined. The presence of financial incentives associated with various production and conservation options invalidate the assumption of profit maximizers acting as if they were minimizing average cost.

In summary there are several factors which imply the need for a more comprehensive policy analysis tool: (1) agricultural soil and water policy has become intertwined with commodity price and farm income support programs; (2) current national policy implementation involves several separate agencies with differing goals and regional scope of interest; (3) national policies have provisions and impacts which vary by state and state policy makers are showing increased interest in soil and water issues; (4) growing surpluses of commodities and excess government budget deficits may bring lower farm product prices and production patterns much different than those observed in the past; (5) a growing realization among policy makers that there are controllable variables available which can be optimized to some degree; and (6) increased computational capacity and theoretical developments in the economic theory area of multi-stage planning and optimization allow more comprehensive approximations to the agricultural economy.

The development of one large model to include all the decision makers and their control variables for current and future soil and water policy analysis is either impossible or prohibitively expensive. Approximations are invariably inconsistent. The goal of this study is not so much to criticize existing models, which serve well for the analysis of many policies, but to show where improvements to meet current and future challenges of policy analysis could be made.

Methods of Approximation

This section of the chapter outlines and critiques some common model types used for natural resource policy analysis. Even though the consistency and behavioral weaknesses are emphasized it must be remembered that there are good reasons for the development and use of these models--better methods are not so readily forthcoming.

Cost minimization simulation of competitive equilibrium

The first type of models discussed are the national cost minimization models developed by Heady and associates starting in the late 1950s (Heady and Srivastava, 1975). Early versions of these models were widely used for investigating regional comparative advantage, excess capacity and commodity program policies. During the last decade versions have been used for soil and water policy analysis with commodity program existence generally assumed away. Cost minimization, profit maximization and equilibrium models originating with Heady have also been applied in policy analysis for developing countries.

These programming models contain activities representing production and consumption in the economy; however, these activities are generally formed as aggregates over individual decision units. The method of aggregating the individual decision units has great influence on how the model ouput should be interpreted. Aggregation in these models involves adding up all the resources of all the farms in each region. The resources assumed to be most limiting or important to the analysis become constraints to a large regional size farm while the other resources are assumed never binding and have their user fees included in each activity objective function coefficient. A production possibilities set is defined to include all the important processes of the region as activities. The resource use and cost coefficients of these activities represent a weighted average of those of the various farms in the region. Note that this specification loses the specialization characteristics of the individual farms. It is implicitly assumed, for consistent aggregation, that all the farms have identical homothetic technology. The region as a whole then exhibits homotheticity. Hence, no farm level results or implications are forthcoming from these models.

The demand levels which the model is constrained to produce are generally obtained as the output of a more aggregated, more general equilibrium type of model. A basic consistency check for these cost minimization models, whether or not the characteristics of the solution satisfy the conditions of the competitive equilibrium solution, generally fails. In particular, the shadow prices on the demand constraints do not match the equilibrium commodity prices in the associated model system which generated the demand levels.

According to Silberberg (1974), in a competitive long run equilibrium firms act as if they are average cost minimizers and hence can be modeled as such. However, most policy analysis cannot be done under the long run equilibrium assumption since the policies being considered involve short run profit incentives and quantity changes are unknown. In addition, the models generally contain enough central institutional constraints that the solution is more like a "national planner's" outcome than one of a general competitive equilibrium nature.

The latest version of these cost minimization models is the current Agricultural Resource Interregional Modeling System (ARIMS) (Robertson et al., 1987). ARIMS is fairly representative of other national resource policy models. ARIMS was developed by the Soil Conservation Service (SCS) and Center for Agricultural and Rural Development (CARD) for the 1980 and 1985 Resource Conservation Act Appraisals. ARIMS is a national inter-regional cost minimization LP model with supporting data sets and some econometric projections of coefficients for future scenarios.

The most limiting aspects of ARIMS for a sector analysis tool are fixed input/product ratios, perfectly in-elastic product demand, perfectly elastic factor supply (except for land and water which are freely available up to a certain point and unavailable thereafter), limited input substitution possibilities via alternative crop rotations and cultural methods, yields increasing due to technology independent of input levels and the assumption of one decision maker with complete knowledge and control (implicitly invoked via the policy goal constraints). Methods to overcome these limitations in LPs exist but the requirement to carry vast quantities of resource and technology detail in ARIMS preclude their use. It is also the standard case with MPs that when commodity price elasticities or factor substitutabilities are modeled the combinatorial dimensions quickly become unmanageable.

Other cost minimization model applications include Taylor et al. (1977), U. S. EPA (1976), Taylor and Swanson (1975) and Rovinsky and Reichelderfer (1979). The outcomes for cost minimization and surplus maximization are compared in Taylor et al. (1977). Cost minimization models contain good resource and technology specification but producer behavior, market environment and policy capacity are weak. For these models a demand vector is usually chosen and held fixed regardless of how much imputed prices change as polices are implemented. Hence, welfare impacts of the policies are biased (Taylor et al., 1977).

Besides the behavioral specification problems, these models cannot be guaranteed to be consistent. The LP aggregation conditions given in the previous chapter are not generally satisfied by empirical data. In general the methods of aggregation chosen have depended more on data availability and ease of coefficient development than on aggregation theorems. An even worse problem occurs where the data collection regions or classifications do not match the model classifications. To overcome this data shortcoming, weighted averages are formed. As stated by Hazell and Norton (1986, page 145): "aggregation bias is always in an upward direction; it overstates resource mobility by enabling farms to combine resources in proportions not available to them individually, and it carries the implicit assumption that all of the aggregated farms have equal access to the same technologies of production." Validity tests have shown ARIMS to be approximately 12 percent more efficient than reality (Robertson et al., 1987). Similar results were found for other large scale programming models (Egbert and Kim, 1975).

Profit maximization

Profit maximization models allow explicit inclusion of policy (and other) distortions expected to cause a non-competitive equilibrium solution (such as commodity subsidies) (Heady and Srivastava, 1975). However, prices are usually held constant and again the welfare impacts are biased just as in the cost minimization case (Taylor et al., 1977). If prices are not held constant via inclusion of demand curves, the implicit assumption is that the producers have monopoly power. Nearly all of the cost minimization model discussion given above with regard to consistency also apply to regional and national profit maximization models. In particular, the method of forming the aggregates is the same as explained for the cost minimization and the same weaknesses occur.

Surplus maximization (or market rent minimization)

Surplus maximization (Takayama and Judge, 1971) or market rent minimization (Smith, 1963) models include price responsive commodity demand and factor supply functions and are capable of determining a spatial equilibrium for prices and quantities (see also Meister et al., 1978). The problem with these models is that resource and technology detail must be given up. The methods of incorporating the price responsive functions include linearization of the relationships and solution as an LP, quadratic programming models under the assumption of linear supply and demand, and price endogenous decomposable models. This type of model can also include non-linear fertilizer response functions, etc., (as can the cost minimization and profit maximization models with the proper construction (linearization) of activities). Aggregates are also formed in the same manner as for the cost and profit models and the same implications for firm level results apply.

The principal difficulties with these NLP models are computational capacity constraints and an overstatement of economic efficiency in some applications. For quadratic models even the largest computers are taxed and computational costs high for models only one-tenth the size needed for the detail of resource policy analysis. In spatial equilibrium applications infinitely small differences in cost cause very large trade flow disruptions. In non-spatial applications flows are unknown, the solution giving only final demand and supply by region.

Further weaknesses of these quadratic market equilibrium models are that the demand and supply functions incorporated must be independent of sector activity, i.e., the model must be partial equilibrium (McCarl and Spreen, 1980) and that the demand and supply functions must be symmetric. These are violated if the sector generates enough income that consumer demand or factor supply functions are shifted. Without integrability (symmetry) the objective function lacks interpretation from a welfare standpoint as the maximization of the sum of producer and consumer surplus (alternatively the minimization of market rent).

With linearization of the surplus terms (Hazell and Norton, 1986) inconsistency is usually introduced. The curse of dimensionality is also encountered and very few models have been specified with more than a few endogenous demand functions and virtually no cross price elasticities or substitutabilities.

Since non-linear models must by nature (computational burden) be more highly aggregated than in the linear case, the consistency errors are likely to be higher. However, in a theoretical example, the opposite outcome was found by Spreen and Takayama (1980). Spreen and Takayama were comparing non-linear functions to linear approximations of the same; their analysis does not give guidelines for deciding on the sorts of questions modelers must answer such as level of regional aggregation, number of distinct producer classes, etc.

Hybrid systems of programming and econometric models

An attempt to capture the good aspects of both national commodity equilibrium econometric models and detailed national interregional LPs in one system is the hybrid model (Huang et al., 1980; Langely et al., 1981). Unfortunately, neither the econometric model nor the LP used were developed specifically for the hybrid. The LP solutions were put into the econometric model which then gave prices to be used in the next iteration of the LP as an equilibrium was sought, and so on.

The LP was a very restrictive fixed coefficients technology similar to ARIMS and so did not really represent the producer responses one would expect given the price changes. The econometric model was time series data based and its supply equations did not correspond to the technology and producer behavioral assumptions of the LP. The amount of calibration work needed for every individual policy analysis application of the system, the artificial flexibility constraints required to achieve result correspondence, and the ad hoc solution adjustment rules to move towards equilibrium limit the usefulness of this approach. The consistency discussion given above for cost minimization and profit maximization models is also applicable here. The econometric model was more highly aggregated than the LP by perhaps a factor of 100 (i.e., one distinct producing unit in the econometric model for each 100 in the LP).

Decomposable model methods

There have been three basic developments in decomposable model optimization relevant to this study: (1) Dantzig and Wolfe (D-W) (1961) decompose a large LP into a main and sub-programs and iterate in a global to sector price directive fashion until the sum of sub-program solutions satisfy global constraints; (2) Kornai and Liptak (K-L) (1965) and Kornai (1973) interpret decomposition as multi-level planning where a central agency iteratively allocates resources to sector planners until resource shadow prices are equated across sectors; (3) Hof and Pickens (H-P) (1986) (who credit Bartlett (1974) and Wong (1980) with the idea) optimize the national forest planning problem by compiling and solving one time a global model containing a designated number of discrete alternative solutions from each sector planning model.

The authors of the D-W and K-L approaches prove convergence while H-P did not. Hof and Pickens perform a number of comparisons between a comprehensive global model and the built up global model for alternative numbers of discrete alternatives and integer versus continuous variable solutions and achieved quite good results. The D-W and K-L approaches are rather rigorous from a mathematical consistency viewpoint whereas H-P are admittedly not. Applications and finer theoretical detail on both the D-W and K-L approaches are given in Goreux and Manne (1973), Kornai (1973), Goreux (1977), and Norton and Schiefer (1980a).

Kutcher (1973) documents the market equilibrium simulating price endogenous decomposable model set up for the CHAC studies in Mexico. This application was essentially using methods developed for decomposable national planning problems. The method was used only for illustration and tests in one region of Mexico since the computer available to the CHAC study was large enough to solve the entire national model without decomposition. The method involved Dantzig-Wolfe (1961) decomposition with demand functions specified at the national level and supply restraints at regional levels. Difficulties were encountered unless composite activities involving several or most commodities were specified at the lower levels since otherwise a particular commodity would come in and out of solution as the system iterated through a sequence of prices.

From an aggregation viewpoint the same sort of consistency discussion given previously for the cost minimization, profit maximization and surplus-equilibrium models applies here. In addition, separability is required for the decomposition to be valid.

The theoretical economic and mathematical conditions needed for feasibility, consistency, and convergence in decentralized planning are given in (Malinvaud, 1967 and 1972). Decomposable algorithms for a non-linear price directive approach are given in (Geoffrian, 1970a). Some detailed model specification and solution algorithms for the iterative approaches of D-W and K-L have been found (Baumol and Fabian, 1964; Geoffrian, 1970a; and Bialas and Karwan, 1984). However, a survey of literature by Norton and Schiefer (1980a) indicate that information exchange has hampered efforts at actual implementation of national multi-level planning model optimization. Also both mathematical and institutional difficulties remain for all of these approaches (Candler et al., 1981; Norton and Schiefer, 1980b). The technical difficulties posed for solution procedures when lower level optimization problems become constraints to upper level decision maker's choices are described in Candler et al. (1981).

For specialized cases of multi-level programming models some solution algorithms have been developed (Bracken and McGill, 1973 and 1974; Bracken et al., 1974).

In a more recent study partitioning methods have been developed so that non-pyramidal hierarchical decision models can be solved with the techniques traditionally used for standard pyramid type structures (Ruefli and Storbeck, 1984). The partitioning methods are used to group the simplex multipliers, or goal deviations, or other coordinate information used in the resource or price directive solution processes. The example application for the technique is in a three (or more) level hierarchical structure where the subordinate level three problems are receiving information (usually goal levels) from two or more superordinate level two problems (rather than from just one). The partitioning consists of simultaneously and independently solving each set of level two unit problems that share coordination of a set of level three units. A set of weights to apply to deviations from the superordinate goals in the subordinate models must then be chosen so that solution can proceed the same as in a standard goal programming application.

The APMAA approach

The Aggregative Programming Model of Australian Agriculture (APMAA) (Walker and Dillon, 1976; and Wicks and Dillon, 1978) utilize linear programming model solutions from many representative farm stratifications as data for econometric estimations of supply response which are then used for policy analysis. Tests and comparisons show good results but a tremendous data collection, management and computational cost is involved. For 500 stratifications and three products with five price steps each, 500 times 125 representative farm solutions were needed.

For this system firm level results are obtainable. Each of the 500 representative farms corresponds closely with a group of actual farms. The results of one of the farm models can be said to simulate the results for any one of the farms in the corresponding groups. In using the system some steps were taken to account for pecuniary externalities and intermediate product trading between firms. Hence, national partial equilibrium results were also given.

Econometrically estimated non-linear programming

A model reported by Frohberg and Fischer (1985) (see also Fischer and Frohberg, 1982), though too highly aggregated to be of use here, contains some concepts relevant to this study. First, the model is an econometrically estimated non-linear programming (NLP) model. A time series data base and econometric model is used iteratively with a NLP production model to arrive at coefficients in the NLP such that a good historical simulation results. Secondly, the production decision process is divided into stages. For example, total fertilizer is first determined for the year. The total fixed fertilizer is then allocated among crops. Here separability, aggregation and two-stage optimization results are used to overcome the fact that total fertilizer data are available by region but not by crop. Finally, some products and inputs are aggregated between stages via price weights.

Endogenous coefficients within a stable basis

A recent proposed programming model improvement includes endogenous coefficients in a decomposed mixed primal-dual approach (Keyzer, 1988). Under the assumption that for a given policy or other model application the basis of optimal activities remains unchanged, imputed constraint prices are used to endogenously optimize over possible values of the activity coefficients. Various potential uses of this technique have been proposed by Keyzer but are still in the development stage. Related to this hierarchical modeling approximations is the possibility of a first stage problem to find the basis followed by second stage problem of adjusting the technical coefficients to get price consistency for resources and other constraints.

National planning and endogenous policy models

Another facet of programming model specification is the proper method of policy optimization and interpretation. The policy analysis problem is really a two-level programming problem, though most researchers do not treat it as such (Candler et al., 1981). Policy makers have control of only a few of the economic variables while individual producers and consumers control the remainder. Individuals take policy parameters as being fixed when making decisions while policy makers take the policy parameter dependent individual response functions as fixed. Due to the mathematical difficulty of setting this all up as a single problem analysts generally specify a model containing only individual decision makers and then parameterize over some "reasonable" values which are available to the policy makers. This approach fails to define clearly the full range of variables under the control of the policy maker and what the optimum choice might be given a policy maker or a societal utility function.

These multi-level alternative decision maker approaches have not been widely applied. The survey of literature by Norton and Schiefer (1980a and 1980b) indicate information exchange has hampered efforts at actual implementation of national multi-level planning model optimization and that both mathematical and institutional difficulties remain for both approaches. The technical difficulties posed for solution procedures when lower level optimization problems become constraints to upper level decision maker's choices are described in Candler et al. (1981).

Goal programming is one method of addressing the endogenous policy or planning problem (for example see Spronk and Veeneklaas, 1983). Levels of outcomes for desirable policy measures are placed simultaneously in the model in a multi-objective programming framework. Efficient sets of outcomes for the policies are generated such that one cannot be improved upon without detracting from one or more of the others. Methods of accomplishing this are outlined briefly in Sposito et al. (1988).

The theoretical economic and mathematical conditions needed for feasibility, consistency, and convergence in decentralized planning have been given (Malinvaud, 1967 and 1972). Decomposable algorithms for a non-linear price directive approach are given in Geoffrian (1970a and 1970b). Some detailed model specification and solution algorithms for the iterative approaches of D-W and K-L have been found (Baumol and Fabian, 1964; Geoffrian, 1970a; and Bialas and Karwan, 1984).

Hierarchical systems

Hierarchical systems of models have been widely used in national and regional planning; these include ad hoc systems of models without theoretically rigorous linkages (Isard and Anselin, 1982; Batten and Andersson, 1983; Isard and Smith, 1983; Liew and Liew, 1984; and Smith, 1986) and large models decomposable into sub-problems (Kornai and Liptak, 1965; Malinvaud, 1967; Goreux and Manne, 1973; Kornai 1975; Goreux, 1977; and Norton and Schiefer, 1980a). The Hof and Pickens (1986) model is a compromise between ad hoc and rigorous approaches.

The use of hierarchical or multi-level planning and modeling for a single producer or by a centralized planning agency has received a lot of attention, particularly in the centrally planned economies. Example studies are Christensen and Obel (1983) and Saaty (1980). Success for market economy based models has been more limited.

Kornai (1973) outlines some difficulties in practical applications of multi-level systems of programming models for planning (see also Candler et al., 1981). Informational and empirical difficulties seem to be enormous for a full scale national implementation of this system (Norton and Schiefer, 1980a).

Consistency through the levels or stages of a hierarchical system and allowable model structure from a solution standpoint are issues that must be addressed in this study. The consistency issue deals primarily with aggregation procedures behavioral simulation while the solvability issue deals with mathematical optimization problems associated with chosen model structures.

Some theoretical work on hierarchical consistency for problems in general have been reported (Auger, 1985a, 1985b, and 1986; Graves, 1982; and Erschler, Fontan, and Merce, 1986). Various technical aspects of setting up and solving hierarchical problems have been investigated (Ward, 1963; Baumol and Fabian, 1964; Weil, 1968; Fisher, 1969; Geoffrian, 1970b; Dyer and Walker, 1982; Bialas and Karwan, 1984; Bard, 1985; Burkard et al., 1985; and Grana and Torrealdea, 1986).

Conclusions

It can be seen that any of the current or past programming models only partly contain the components of an ideal model. Expanding the LP models to more fully simulate the agricultural sector is impossible due to computational constraints. Other possibilities such as quadratic programming, self-dual models, hybrid LP and econometric, and pure econometric models also face such constraints and only adequately meet some of the ideal criteria at the expense of others.

The existing literature on aggregation methods of overcoming these capacity and data constraints generally require such strict empirical conditions or assumptions as to render the analysis invalid. As has been shown in the literature cited in this chapter most decomposable or multi-level approaches also rely on strict aggregation or separability conditions which are not supported by empirical observation. There is a scarcity of literature on the magnitude of analysis error when applied methods ignore the strict conditions. In summary, though these single or informally linked models with their limitations may have been appropriate for past analyses (and still do a good job for many current policy questions) there are several factors which imply the need for a more comprehensive policy analysis tool.

This review of ideal sector model approximations indicate a wide range of behavioral and aggregation specifications. A wide range of models have been used for policy analysis because there have been a wide range of questions needing answers and available resources for analysts to work with. A comprehensive fully ideal sector model is not needed for all policy questions. As an example, if a fertilizer tax is expected to only alter cultivational and rotational on-farm practices with no impact on either yields or marginal costs, a partial equilibrium model is not needed. On the other hand a comparison of the trade deficit effects of two alternative export enhancement policies need not include a detailed analysis of soil erosion impacts by land class, etc.

CHAPTER IV.

ILLUSTRATIVE EXAMPLES OF INTER-MODEL CORRESPONDENCES AND CONSISTENCY

Simple analytical models are used to illustrate appropriate inter-model behavioral correspondences for maximizing consistency between regional and national models. The aggregation implications implicit in the examples are discussed in terms of exact aggregation theorems and likely empirical facts of model building. From these illustrations the implications of either seeking or assuming aggregation consistency are shown and a set of guidelines for constructing natural resource policy models are derived.

Preliminary results established in earlier chapters indicate that a fully consistent set of national and regional models satisfying all the aggregation theorems may not be feasible. Mathematically and theoretically such a system is possible; however, as shown in previous chapters and below the requirements involve such untenable restrictions on the microeconomics of producer and consumer behavior as to be unacceptable. If empirical observations were found to support such restrictions the resulting modeling system would be unuseful and uninteresting from a policy analysis viewpoint due to the homotheticity requirements of most of the theorems.

In various modeling contexts researchers have developed methods of achieving fairly accurate results by either relaxing the stringent aggregation conditions presented in the previous chapters or assuming they are satisfied. However, the methods of relaxation to maximize consistency are tightly connected to the characteristics of the specific problem being modeled (this point is explained in more detail via the examples of this chapter); assuming the aggregation consistency conditions are met causes other analytical problems.

Unfortunately, there seem to be no general rules for such "near-consistent" modeling efforts. What is needed for consistency varies case by case and either you have consistency or you do not. For a given problem one can explore the empirical error implications of inconsistent aggregation; however, as parameters of the problem change the form of the error also changes making it rather intractable. For specific model types, such as for LP models as discussed in the previous chapter, various guidelines have been proposed, but they are also based on empirical data.

If the dimensionality of the problem is reduced in aggregation, such as for example looking only at aggregate output and not inputs, the consistency conditions are relaxed only a little. However, one must be willing to accept indices of output rather than output and the restrictions on functional form are still rather severe. Disaggregation of the aggregate indices also requires restrictive assumptions. Most available data have not generally been collected and processed in the appropriate index forms for the non-linear aggregators and technology functional forms.

The national and regional models used in these illustrations may, for the same area, involve different levels of aggregation and contain differing aggregations across individual areas or categories. Aggregate products and inputs are also defined differently in some instances. The question addressed here is how to structure the national and regional models so they are internally consistent and so that they support each other's results when used for analysis of the same policy question. Though these models are much simpler than actual models used in policy analysis, their simplicity allows derivation of analytical results which can provide guidelines for the construction of more complex models.

The particular system arrangement of models used in these examples are somewhat restrictive. However, they correspond to some commonly used methods. A national cost minimization model with several producing regions is compared to a set of profit maximizing regional models. Initially there is one regional model for each national model region but that is changed in some later examples. The national model chooses the production pattern to meet specified demand at the least cost. It is assumed that this national criterion simulates the long run minimum cost competitive equilibrium of a nation of profit maximizing producers and utility maximizing consumers.

The equilibrium marginal costs of production in the national model in the optimal solution, with some adjustments, become regional prices for the profit maximization models. In turn, the base acreage (and other fixed outcomes) of the regional models associated with the profit incentives of agricultural policy become artificial constraints in the national model. Analytical derivation provides economic interpretations to these price and constraint correspondences.

The national and regional model outcomes will be illustrated for several cases. However, first considerable discussion is needed to establish the economics of the models. This involves the properties assumed for the aggregated and individual technology, resources and criterion functions and the assumptions underlying aggregation over individual agents in both the national and state models. Also included are the mathematical conditions guaranteeing existence of optimal solutions.

Once the models are fully described then cases of increasing complexity are shown. These cases gradually develop the results needed for the guidelines of constructing models and systems in a more consistent manner.

Technological and Terminology Preliminaries

Before proceeding with specification and interpretation of example models some underlying assumptions and restrictions are given. These are the assumptions and restrictions that ensure the microeconomic behavioral axioms are satisfied and that guarantee the existence of unique solutions to the various problems which will be optimized (this notation is from Varian, 1984). In the following discussion, particularly for the supply side of the models, the word "firm" is frequently used to describe the decision taking agents. In some cases "firm" will refer to individual agents directly and in other cases such as the regional profit maximization models it will refer to an aggregate over several firms.

Assume the firm has n possible goods to serve as inputs and/or outputs. Denote a specific production plan involving these goods as a vector y in R^n where $y_i > (<) 0$ for a net output (input) for $i = 1,...,n$. The set of all feasible netput vectors, the production possibilities set, is denoted Y and is also a subset of R^n. Given that in the short run some factors of production may have restricted use, a restricted or short run production possibilities set Y(z), where z is the parameter vector of restrictions, may be defined.

If attention is restricted to one output and several inputs (as it is in these results) the following concepts are defined:

$V(y) = \{x \text{ in } R^n: (y,-x) \text{ is in } Y\}$ as input requirement set,

$Q(y) = \{x \text{ in } R^n: x \text{ is in } V(y), x \text{ is not in } V(y') \text{ for } y'>y\}$ as an isoquant,

$Y(z) = \{(y,-q,-k) \text{ in } Y: k = z\}$ as the restricted production possibilities set, and

$f(x) = \{y \text{ in } R: y \text{ is the maximum output associated with } -x \text{ in } Y\}$ as the production function.

Defining the assumptions and/or restrictions on the input requirement sets shows the results needed for the technology representation. The input requirement set is said to be Regular and to exhibit Monotonicity and Convexity.

Regularity means that V(y) is non-empty, $y > 0$ implies 0 is not in V(y) and V(y) is a closed subset of Rn. In words this means that there is some way to produce any level of output, something cannot be produced from nothing and if there is a sequence (x^i) of input bundles that can each produce y, and if (x^i) converges to x^* then x^* can also produce y.

Monotonicity implies that free disposal of excess inputs is available. If x is in V(y) and $x' > x$, then x' is also in V(y).

Convexity requires that if x is in V(y) and x' is also in V(y) then $tx + (1-t)x'$ is in V(y) for all $0 < t < 1$. In addition, if Y is a convex set, then V(y) is a convex set, V(y) is a convex set if and only if $f(x)$ is a quasiconcave function, and if $V(y)$ is a convex set, then Y need not be convex. Convexity allows flat spots on isoquants but strict convexity requires that isoquants be strictly rotund.

The production function, $f(x)$, is the underlying base of the illustrative models of this chapter. Chambers (1988, page 9) lists the desirable properties of a production function as:

1. a. if $x' \geq x$, then $f(x') \geq f(x)$ (monotonicity);
   b. if $x' > x$, then $f(x') > f(x)$ (strict monotonicity);
2. a. $V(y) = \{x: f(x) > y\}$ is a convex set (quasi-concavity);
   b. $f(x^o + (1-t)x^*) > tf(x^o) + (1-t)f(x^*)$ for $0<t<1$ (concavity);
3. a. $f(0_n) = 0$, where 0_n is the null vector (weak essentiality);
   b. $f(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_n) = 0$ for all x_i (strict essentiality)
4. the input requirement set $V(y)$ is closed and non-empty for all $y > 0$ (no free output);
5. $f(x)$ is finite, non-negative, real valued, and single valued for all non-negative and finite x;
6. a. $f(x)$ is everywhere continuous; and
   b. $f(x)$ is everywhere twice-continuously differentiable.

For an individual firm the decision to maximize profits is written as maximize $P = p*y - \sum w_i x_i$ where $y = f(x)$. For any level of output the firm will want to produce at a minimum cost so a dual problem can be written as minimize $C = \sum w_i x_i$ subject to $f(x) = y$ and $y = y^*$. These decision problems can be solved analytically to get a cost function $C = c(y,w)$. In the problems worked out in this chapter extensive use is made of the cost function; the production function and input quantities do not always appear explicitly in the optimization problem.

When the technology and optimizing rules as defined above are combined in a mathematical formula, different terminology is used to describe the further requirements that must be met for (1) a solution to exist and (2) for a guarantee that the solution found is the global optimum. Define the Hessian matrix for an unconstrained optimization problem as consisting of the second order partial derivatives of $f(x)$. For a maximum (minimum) H must be negative definite (positive definite) (Chiang, 1974, page 351). For a constrained optimization problem a bordered Hessian contains the second partial derivatives of the Lagrangian; for a maximum (minimum) the bordered Hessian must again be negative definite (positive definite) (Chiang, 1974, page 389).

The cost function resulting from the optimization problem given the technology described above also has several important properties (Chambers, 1988, page 52):

1. $C(w,y) > 0$ for $w > 0$ and $y > 0$ (non-negativity);
2. if $w' > w$ then $C(w',y) > C(w,y)$ (non-decreasing in w):
3. concave and continuous in w;
4. $C(tw,y) = tC(w,y)$ for $t > 0$ (positively linearly homogeneous);
5. if $y > y'$ then $C(w,y) > C(w,y')$ (non-decreasing in y); and
6. $C(w,0) = 0$ (no fixed costs).

In addition to the conditions given here to guarantee an optimum, the presence of inequality constraints in the illustrative examples require the use of Kuhn-Tucker conditions to augment the standard FOC. The Kuhn-Tucker conditions will be described at the point where they first come up in the models.

The Basic Model Set

This section includes a technical description of the models and correspondences to be used in the various illustrative cases of this chapter. The problem at the national level is to minimize the total cost of production for a specified level of demand. For illustrative purposes "resource use flexibility" constraints in the national model reflecting the distortions caused by commodity subsidies in the regional profit models are also included. These constraints can also represent other types of limits to behavior or other institutional arrangements and the interpretations derived in this chapter are similarly applicable.

For simplicity it is assumed that the content of economic activity in each region is adequately representable by individual cost functions which are separable from each other both within and across regions. The only multi-product supply consideration included is where each commodity within a region compete for the scarce resources of that region. It is assumed that the micro or firm level technology underlying these functions meet all the standard assumptions outlined above. At this point in the analysis no explicit assumptions or statements are made about aggregation methods. It is assumed that whatever firms exist in each region were somehow aggregated to arrive at the cost functions.

This section proceeds without consideration of whether or not the aggregate functions satisfy the micro level technology and cost assumptions outlined above. The problem is specified as in (4.1). In (4.1.i) resource use is constrained by the fixed amount available; in (4.1.ii) the commodity demands to be met are given; and in (4.1.iii) the resource use for specific commodity (flexibility) constraints are specified. The entire problem in (4.1) is converted to commodity units rather than resource units by the inverse yield coefficients, a_{ij}.

$$(4.1) \quad \min_{Q_{ij}} C = \sum_{i=1}^{I} \sum_{j=1}^{J} C_{ij}(Q_{ij}, r_i)$$

subject to:

(i) $\sum_{j=1}^{J} a_{ij}Q_{ij} \leq L_i$ for $i = 1,\ldots,I$

(ii) $\sum_{i=1}^{I} Q_{ij} \geq Q_j$ for $j = 1,\ldots,J$

(iii) $a_{ij}Q_{ij} \geq L_{ij}$ for $i = 1,\ldots,I$ and $j = 1,\ldots,J$

(iv) $Q_{ij} \geq 0$ for $i = 1,\ldots,I$ and $j = 1,\ldots,J$

where $C_{ij}(Q_{ij}, r_i)$ is the cost function of jth commmodity in ith region given the vector r_i of input prices;

a_{ij} is the resource requirement per unit of commodity, i.e., the inverse of per acre yield;

L_i is the resource availability in region i;

L_{ij} is the flexibility constraint on the use of resource i in producing commodity j; and

Q_j is the final demand for jth commodity.

$$(4.2)\quad \text{Min } L = \sum_{i=1}^{I} \sum_{j=1}^{J} C_{ij}(Q_{ij}, r_i) + \sum_{i=1}^{I} u_i \Big(- \sum_{j=1}^{J} a_{ij}Q_{ij} + L_i\Big)$$

$$+ \sum_{j=1}^{J} \lambda_j \Big(- \sum_{i=1}^{I} Q_{ij} + Q_j\Big) + \sum_{i=1}^{I} \sum_{j=1}^{J} \gamma(-a_{ij}Q_{ij} + L_{ij})$$

FOC: (i) $$\frac{\partial L}{\partial Q_{ij}} = MC_{ij} - u_i a_{ij} - \lambda_j - \gamma_{ij} a_{ij} \geq 0; \quad Q^*_{ij} \frac{\partial L}{\partial Q_{ij}} = 0;$$

$$Q^*_{ij} \geq 0 \text{ for } i = 1,\ldots,I \text{ and } j = 1,\ldots,J$$

(ii) $$\frac{\partial L}{\partial u_i} = - \sum_{j=1}^{J} a_{ij}Q_{ij} + L_i > 0; \quad u^*_i \frac{\partial L}{\partial u_i} = 0;$$

$$u^*_i \leq 0 \text{ for } i = 1,\ldots,I$$

(iii) $$\frac{\partial L}{\partial \lambda_j} = Q_j - \sum_{i=1}^{I} Q_{ij} \geq 0; \quad \lambda^*_j \frac{\partial L}{\partial \lambda_j} = 0;$$

$$\lambda^*_j \geq 0 \text{ for } j = 1,\ldots,J$$

(iv) $$\frac{\partial L}{\partial \gamma_{ij}} = L_{ij} - a_{ij}Q_{ij} \geq 0; \quad \gamma^*_{ij} \frac{\partial L}{\partial \gamma_{ij}} = 0;$$

$$\gamma^*_{ij} \geq 0 \text{ for } i = 1,\ldots,I \text{ and } j = 1,\ldots,J$$

where u_i is the imputed price of resource unit i;

λ_j is the imputed price of commodity j; and

γ_{ij} is the imputed "rent" to the constraints that specify use of resource i for j.

$$(4.3)\quad \text{(i)}\quad u^*_i = \frac{\partial L}{\partial L_i} = \frac{\text{cost}}{\text{input}_i} \leq 0; \quad (-u^*_i) \text{ is resource}_i \text{ "rent"}$$

$$\text{(ii)}\quad \lambda^*_j = \frac{\partial L}{\partial Q_j} = \frac{\text{cost}}{\text{demand}_j} \geq 0; \quad \lambda^*_j \text{ is commodity}_j \text{ price}$$

$$\text{(iii)}\quad \gamma^*_{ij} = \frac{\partial L}{\partial L_{ij}} = \frac{\text{cost}}{\text{flex}_{ij}} > 0; \quad \gamma^*_{ij} \text{ is per unit}$$

(of resource) cost of forcing non-optimal use

The problem in (4.1) can be specified in Lagrangian form as in (4.2) and solved with standard analytical techniques. Note that the inequalities of (4.1) and (4.2) are handled in the FOC of (4.2) with the Kuhn-Tucker complementary slackness conditions. Use of the envelope theorem shows that (Silberberg 1978) concrete interpretations are available for the Lagrangian multipliers of (4.2) as shown in (4.3).

The Kuhn-Tucker formulation of the FOC for an optimum account for the presence of inequality constraints and for the possibilities of corner solutions (such as will occur in the linear case presented below). As shown in (4.2.ii) and (4.3.i) the resource only receives an imputed rent if it is in short supply. The possibility of marginal cost of production exceeding commodity value such that a corner solution of zero production would occur is allowed for each commodity in each region by (4.2.i) while (4.3.ii) show that if any commodity is overproduced its imputed shadow price will be zero. Finally, the possibility that a flexibility constraint might be more binding than a resource availability constraint is accounted for in (4.2.iv) and (4.3.iii).

The equilibrium conditions implied by satisfaction of the first order conditions of (4.2) are shown in (4.4). In general, these equilibrium conditions are obtained by assuming the FOC are satisfied as equalities with right hand side values of zero.

They can then be set equal to each other, used in ratios, or have common elements factored out.

Condition (4.4.i) gives the allocation of the fixed resource in each region as shown in Figure 1 where the marginal rate of return on the resource is equalized across all uses. In the case shown it is assumed that a distorting flexibility (resource use) restriction is binding, $\gamma_{ij} > 0$, and rents are reduced. For this to hold strictly for all possible comparisons it must be the case that the FOC do hold as equalities. In the linear MC case shown below equalities do not hold for any values of the endogenous variable. Therefore, no equilibrium can be defined. Condition (4.4.ii) shows that given a national demand for a commodity, the marginal cost of producing the good must be equalized across regions. This is illustrated in Figure 2 along with the competing impacts of the resource and flexibility rents. Condition (4.4.iii) and Figure 3 show that the ratio of implied marginal values of any two goods must be constant across all regions.

The regional profit maximization problems corresponding to (4.2) are given in (4.5) with the FOC in (4.6). For this initial model setup the desired analytical correspondences between the regional and national models are shown in (4.7).

(4.4) (i) $(1/a_{ij})[MC_{ij} - \lambda_j - \gamma_{ij}a_{ij}] = (1/a_{ij})[MC_{iz} - \lambda_z - \gamma_{iz}a_{iz}]$

for any commodity pair j,z = 1,...,J (j = z) in any region i = 1,...,I

(ii) $MC_{kj} - (u_k + \gamma_{kj})a_{ij} = MC_{pj} - (u_p + \gamma_{pj})\, a_{pj}$

for every commodity j = 1,...,J across any pair of regions k,p = 1,...,I (k = p)

(iii)
$$\frac{MC_{kj} - (u_k + \gamma_{kj})a_{kj}}{MC_{kz} - (u_k + \gamma_{kz})a_{kz}} = \frac{\lambda^{*}j}{\lambda^{*}z} = \frac{MC_{pj} - (u_p + \gamma_{pj})a_{pj}}{MC_{pz} - (u_p + \gamma_{pz})a_{pz}}$$

as a ratio for any pair of goods j,z = 1,...,J (j = z) and across all pairs of regions k,p = 1,...,I for k = p

(4.5)
$$\underset{Q_{ij}}{\text{Max}}\ \pi^{i} = \overset{J}{\underset{j=1}{}} (P_{ij}Q_{ij} - C_{ij}(Q_{ij}) - r_i a_{ij}Q_{ij}) \quad (i = 1,\ldots,I)$$

where π_i is profit for ith region;

P_{ij} is "farm gate" price in i for jth good; and

r_i is the resource use charge (rental to reflect opportunity cost).

(4.6) (i)
$$\frac{\partial \pi_i}{\partial Q_{ij}} = P_{ij} - MC_{ij} - r_i a_{ij} < 0;\ Q^{*}_{ij}\ \frac{\partial \pi_i}{\partial Q_{ij}} = 0;\quad Q^{*}_{ij} > 0$$

(ii)
$$\frac{\partial \pi_i}{\partial Q_{ij}} = (P_{ij} + s_{ij}) - MC_{ij} - r_i a_{ij} < 0;$$
$$Q^{*}_{ij}\ \frac{\partial \pi_i}{\partial Q_{ij}} = 0;\ Q^{*}_{ij} > 0$$

(4.7) (i) $P_{ij} = \lambda_j$

(ii) $s_{ij} = \gamma_{ij}a_{ij}$

(iii) $-r_i a_{ij} = u_i a_{ij}$

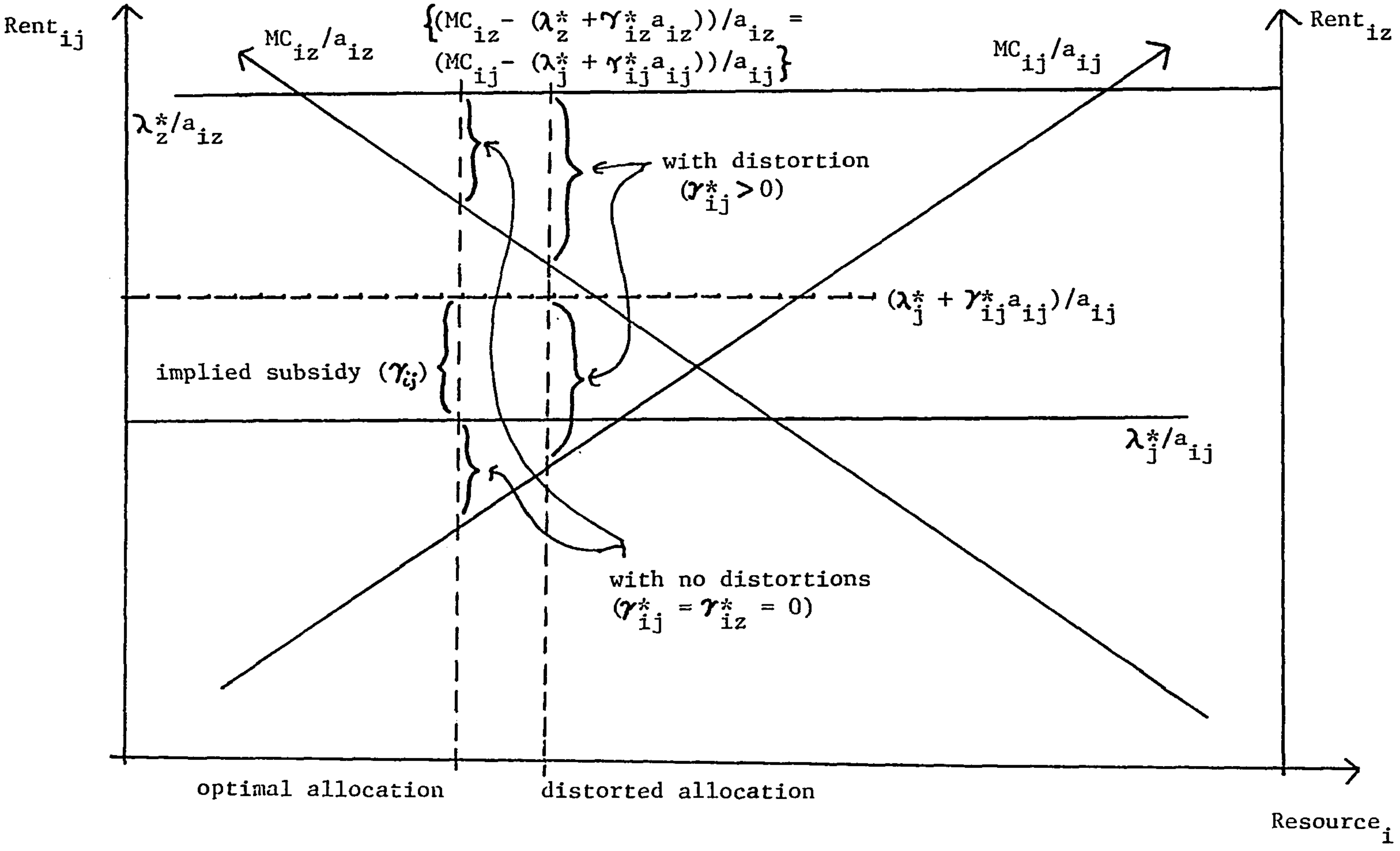


Figure 1. Equalization of rent to a fixed factor across all uses

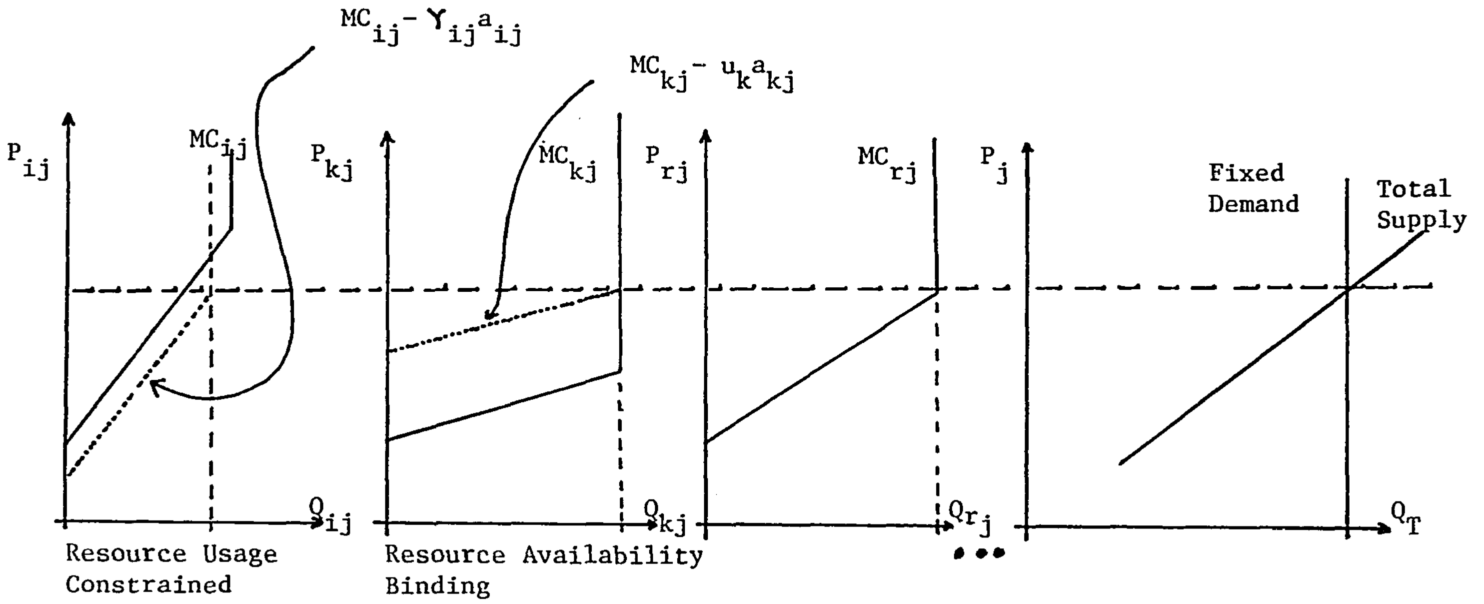


Figure 2. Equalization of the marginal cost of production of a commodity across regions

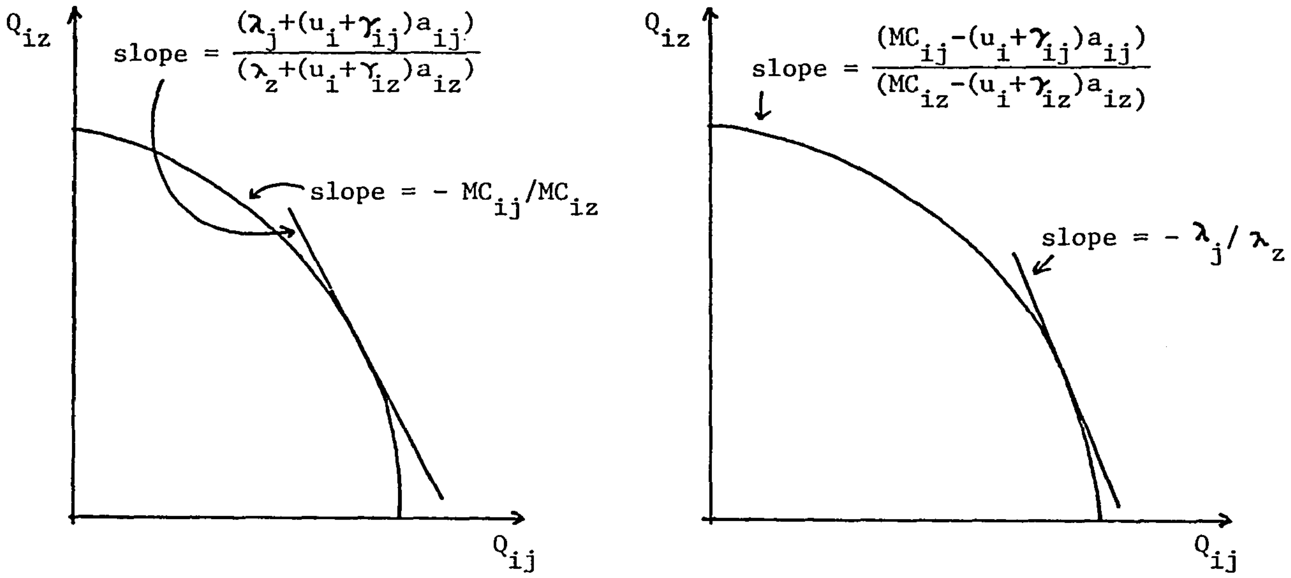


Figure 3. Marginal value ratios for commodities equalized across regions

As (4.7.i) shows the shadow prices (solution value marginal costs of production) found as part of the national model competitive equilibrium must be used as product prices in the regional profit models. The presence of subsidies which result in the profit maximization model outcome being distorted from the national cost minimization competitive equilibrium can be interpreted as shown in (4.7.ii). This requires that the national model have resource use flexibility constraints equivalent to the resource use outcomes of the state models. Alternatively, the pricing correspondence between the level of artificial resource use flexibility constraints of the national models and commodity subsidies allow the national solution to be imposed on the regional profit maximization models and vice versa.

Resource use costs must be included in the regional profit models to account for the opportunity costs of using the resource in the alternative commodities producible in the region. Unfortunately, these resource rents depend on the national solution which implies a national solution is needed before the state model specification process can proceed. The worst implication is that no policy analysis with one or several regional models can proceed unless it is certain the policy will not cause a change in the previously found rental value. It is also implied that if the policy involves profit incentives the national solution should not be attempted until the regional profit model outcomes are available.

With these non-linear, well behaved cost functions at the aggregate level inter-model behavioral consistency is achieved but nothing has been said about the requirements for aggregation consistency. The set of homothetic non-linear functions satisfying aggregation consistency is small and data may not be available in the form of the required aggregates for them.

Linear Aggregation Versus Behavioral Consistency

The aggregation consistency conditions yielding industry cost and output as the sum of the same for firms being aggregated results in behavioral inconsistency between national and regional models. With linear aggregation rules consistency only occurs if every firm has identical constant marginal cost. Aggregation over identical firms in each region results in constant marginal cost at the regional level. With those results the equilibrium FOC conditions of (4.2) break down and there are no inter-model price correspondences such that regional model outcomes are equivalent to the national; only fixed output level restrictions would give consistency. Fixed output correspondences conflict with standard behavioral assumptions of the underlying firms and so cannot be used. These implications also apply to profit maximizations models when used for a region or a nation without consideration of competitive equilibrium outcomes.

The case of constant marginal costs is important not just because it guarantees aggregation consistency but also because it corresponds closely to the actual linear programming (LP) models used as approximations in policy analysis. Whether or not the issue of aggregation consistency over micro units is explicitly addressed, those LP models contain activities which are each linear in inputs and outputs and the entire model is homogeneous of degree one as a unit. Within an LP model, through the appropriate use of activities and constraints, various non-linear behavior can be simulated. However, it is still the case that at the margin, as in sensitivity analysis, the LP solution may exhibit linearity for some large parametric variation before the variables in the basis change. This implies that at least locally, as in a large enough neighborhood of an optimal solution that policy impacts do not change the basis variables, or in a global sense the supply functions obtained parametrically from the LP (which implicitly represent MC schedules) are linear. In addition the aggregators used to construct regional, national and sector models are generally linear or Leontief, both of which give consistent aggregation. The actual aggregator and analytical model choice is greatly constrained by the manner in which the primary data have been collected and aggregated, which is usually with simple or weighted linear rules.

Several cases of differing complexity are considered. First, the case of each regional model's technology and constraint set being the result of exact decomposition of the technology and constraints of the national model along regional lines is considered. Secondly, the additional complications arising from the regional profit maximization models corresponding to weighted portions of the regions of the national model are investigated. For the first case it is initially assumed that the flexibility constraints (as in (4.1.iii)) are not binding; the additional complications when the flexibility constraints are binding (and are also consistent with resource availability) is then shown.

<u>Identical regions and non-binding resource use flexibilities</u>

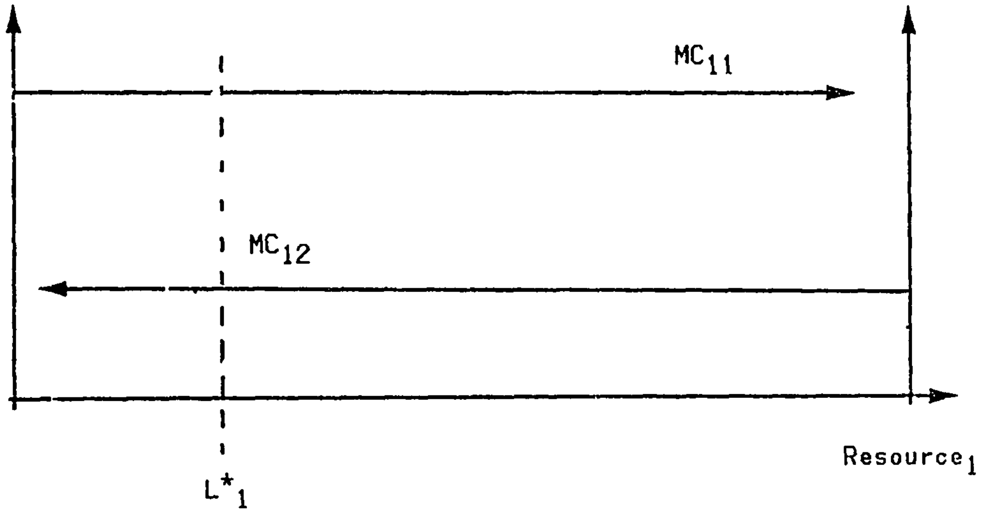
Assume that the national model is as specified in (4.1) and (4.2). For notational simplicity the r_i vector of input prices has been dropped since no comparative statics with it will be done. Assume that the conditions for consistent aggregation from the firm level to the regional level aggregate model representations have been met such that each MC_{ij} is a constant. It is shown below that the equilibrium conditions (4.4.i) will not necessarily be satisfied as equalities since the LHS and RHS components may be inequalities. It is still the case that the u_i term (resource rent) can be factored out; whether the LHS is greater than, equal to or less than the RHS is an empirical question.

Proceeding with this qualitative analysis requires some assumptions about relative marginal cost of production of the commodities in regions. The arbitrary assumptions made here are shown in Figure 4 with the assumption that each γ_{ij} is equal to zero. The explanation of the model solution implied by Figure 4 also shows why (4.4.i) do not hold as equalities in the linear cost case.

Figure 4 is drawn under the assumption that MC_{12} is much lower than all other MC_{ij} and with region 2 having overall relatively higher MC. It is also assumed that the resource flexibility constraints are non-binding and that more than enough resources exist to produce all specified demands.

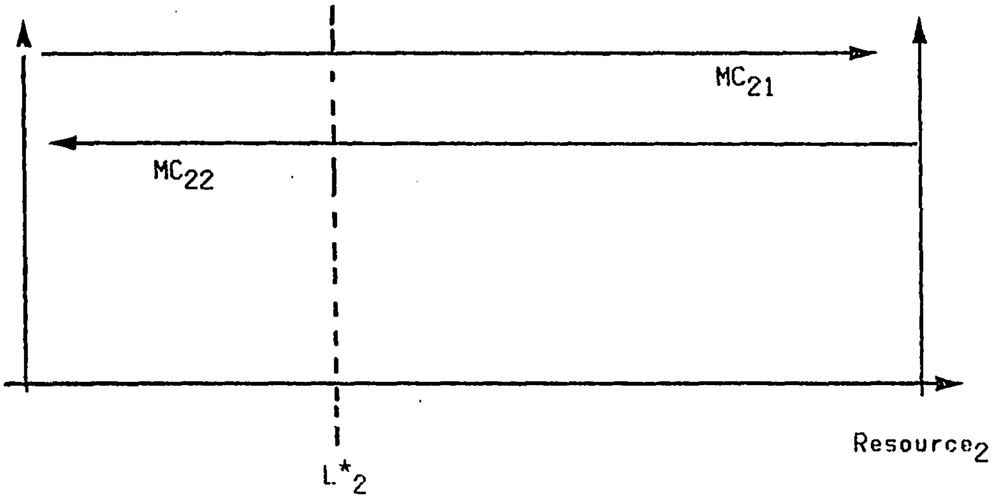
With these assumptions, resource 1 is used as necessary to produce all of Q_2. Any remaining L_1 is used in production of Q_1. Only if region 1 is unable to produce all of both Q_1 and Q_2 will any of the resources in region 2 be utilized. The illustration assumes that some of the resources of region 2 are used for production of commodity 1. The implied resource rents and commodity values can also be seen in Figure 4.

Since having one less unit of resource 1 available requires shifting production of $1/a_{11}$ units of commodity 1 from region 1 to region 2, the resource value of region 1 is equal to



$Q_2 = (L_1 - L^*_1)/a_{12} = Q^*_{12}$

$Q^*_{11} = L^*_1/a_{11}$



$Q^*_{21} = (Q_1 - Q^*_{11})L^*_2/a_{12}$
$Q^*_{22} = 0$

Figure 4. Resource allocation in the linear marginal cost case

$(1/a_{11})[MC_{21}-MC_{11}]$. The marginal value of good 1 is MC_{21} since that is the cost of producing an additional unit. Derivation of the marginal value of good 2 requires consideration of the opportunity cost of resource use in region 1 and the relative resource use requirements of the commodities.

Production of an additional unit of good 2 requires a_{12} units of resource 1. Using a_{12} units of resource 1 for good 2 makes that much less available for good 1. Some production of good 1 will be shifted into region 2 at a cost increase per unit of commodity of $(MC_{21}-MC_{11})$. If $a_{12} \lesseqgtr a_{11}$ then a shift of $\lesseqgtr$ one unit of good 1 to region 2 will occur. This implies that the marginal cost of producing an additional unit of good 2 is $(a_{12}/a_{11})MC_{21}$.

This equilibrium information can be used to determine the outcome of conditions (4.4.1). Note that each γ_{ij} equal zero while $(MC_{11}-\lambda_1) < 0$ since $\lambda_1 = MC_{21}$ and it is assumed that $MC_{21} > MC_{11}$. Since $Q^*_{12} > 0$, $(MC_{12}-\lambda_2) = 0$ where $\lambda_2 = (a_{12}/a_{11})MC_{21}$. Hence, for region 1 (4.4.i) is not an equality. The same type of result can be shown for region 2. The regional profit maximization analysis given below indicates that there is really not an equilibrium for this system of models unless profit maximizers face perfectly inelastic regional demand curves which are consistent with their technology and other constraints.

The price sets for the regional profit maximization models implied by the national model outcome are shown in (4.8) where it is assumed that the absence of resource use flexibility constraints corresponds with the lack of subsidies (see the form of price correspondences given in (4.7)). The profit maximization models, given these price sets, are shown in (4.9) and (4.10) along with their FOC.

In region 1 the equilibrium given by (4.9.i) and (4.9.ii) is not well defined. The actual outcome depends on whether or not producers charge themselves the resource use opportunity cost. If the resource use charge is not included the profits per unit for commodity 1 are positive ($MC_{21} > MC_{11}$) while for good 2 positive profits or not is an empirical question [$(a_{12}/a_{11})MC_{21} \gtrless MC_{12}$?].

Which ever good has the largest per unit profits will receive the entire allocation of the resource and its production level will be $Q^*_{ij} = L_i/a_{ij}$. If both goods have positive profits that are equal then the solution is indeterminate.

If a resource use charge $(MC_{21} - MC_{11})/a_{11}$ is included in region 1 then profits for commodity 1 are exactly equal to zero while for good 2 it is an empirical question. Either one good receives the entire resource allocation, neither are produced at all or else the solution is indeterminate if both have equal positive profits.

(4.8) (i) $[P_{11}, P_{12}, r_1, s_{11}/a_{11}, s_{12}/a_{12}] =$

$$[MC_{21}, (a_{12}/a_{11})MC_{21}, (MC_{21}-MC_{11})/a_{11}, 0, 0]$$

(ii) $[P_{21}, P_{21}, r_2, s_{21}/a_{21}, s_{22}/a_{22}] =$

$$[MC_{21}, (a_{12}/a_{11})MC_{21}, 0, 0, 0]$$

(4.9) (i) $$\max_{Q_{ij}} \pi_1 = P_{11}Q_{11} - C(Q_{11}) + P_{12}Q_{12} - C(Q_{12}) - r_1L_1$$

(ii) $$\frac{\partial \pi_1}{\partial Q_{11}} = MC_{21} - MC_{11} - (MC_{21} - MC_{11}) \leq 0;\ Q^*_{11} \frac{\partial \pi_1}{\partial Q_{11}} = 0;$$
$$Q^*_{11} \geq 0$$

(iii) $$\frac{\partial \pi_2}{\partial Q_{12}} = MC_{21} - MC_{12} - (a_{12}/a_{11})(MC_{21} - MC_{11}) \leq 0;$$
$$Q^*_{12} \frac{\partial \pi_2}{\partial Q_{12}} = 0;\ Q^*_{12} \geq 0$$

(4.10) (i) $$\max_{Q_{ij}} \pi_2 = P_{21}Q_{21} - C(Q_{21}) + P_{22}Q_{22} - C(Q_{22}) - r_2L_2$$

(ii) $$\frac{\partial \pi_2}{\partial Q_{21}} = MC_{21} - MC_{21} \leq 0;\ Q^*_{21} \frac{\partial \pi_2}{\partial Q_{21}} = 0;\ Q^*_{21} \geq 0$$

(iii) $$\frac{\partial \pi_2}{\partial Q_{22}} = (a_{12}/a_{11})MC_{21} - MC_{22} \leq 0;\ Q^*_{22} \frac{\partial \pi_2}{\partial Q_{22}} = 0;$$
$$Q^*_{22} \geq 0$$

The same type of outcome exists for region 2. There are no derivable intermodel price correspondences which will ensure behavioral consistency. In conclusion, the linear cost conditions needed for aggregation consistency result in inconsistent behavioral outcomes among the models.

Identical regions, all resource use restrictions binding

Assume model parameters the same as in the previous case except that a full set of resource use restrictions, L_{ij}, are now binding. Assume that these restrictions give outcomes different than the optimal ones found in the previous case, that the restrictions are consistent with resource availability, and that free disposal of any excess production is possible. With these assumptions it is known before the solution is obtained that $u_1 = u_2 = {}_1 = {}_2 = 0$. The solution at the national level model is determined as $Q^*_{ij} = L_{ij}/a_{ij}$ since all marginal costs are strictly positive and the resource use restrictions will hold as equalities.

In the regional profit maximization models the implied commodity prices must account for the flexibility constraints as shown in (4.7.ii), i.e., in the form of commodity subsidies. This interpretation assumes the "imputed rent" on the flexibility constraints is equivalent to the subsidies on commodities in the profit maximization models when adjusted to equal units. The

(4.11) (i) $[P_{11},P_{12},r_1,s_{11}/a_{11},s_{12}/a_{12}] =$

$[0,0,0,MC_{11}/a_{11},MC_{12}/a_{12}]$

(ii) $[P_{21},P_{21},r_2,s_{21}/a_{21},s_{22}/a_{22}] =$

$[0,0,0,MC_{21}/a_{21},MC_{22}/a_{22}]$

(4.12) (i) $\underset{Q_{ij}}{\text{Max}}\ \pi_1 = MC_{11}Q_{11}/a_{11} - C(Q_{11}) + MC_{12}Q_{12}/a_{12} - C(Q_{12})$

(ii) $\frac{\partial \pi_1}{\partial Q_{11}} = MC_{21}/a_{11} - MC_{11} \leq 0;\ Q^*_{11}\frac{\partial \pi_1}{\partial Q_{11}} = 0;\ Q^*_{11} \geq 0$

(iii) $\frac{\partial \pi_1}{\partial Q_{12}} = MC_{12}/a_{12} - MC_{12} \leq 0;\ Q^*_{12}\frac{\partial \pi 1}{\partial Q_{12}} = 0;\ Q^*_{12} \geq 0$

(4.13) (i) $\underset{Q_{ij}}{\text{Max}}\ \pi_2 = MC_{21}Q_{21}/a_{21} - C(Q_{21}) + MC_{22}Q_{22}/a_{22} - C(Q_{22})$

(ii) $\frac{\partial \pi_2}{\partial Q_{21}} = MC_{21}/a_{21} - MC_{21} \leq 0;\ Q^*_{21}\frac{\partial \pi 2}{\partial Q_{21}} = 0;\ Q^*_{21} \geq 0$

(iii) $\frac{\partial \pi_2}{\partial Q_{22}} = MC_{22}/a_{22} - MC_{22} \leq 0;\ Q^*_{22}\frac{\partial \pi\ 2}{\partial Q_{22}} = 0;\ Q^*_{22} \geq 0$

(4.14) (i) $4 - \lambda_1 + u_1 \geq 0;\ Q*_{11}\ (\ ) = 0;\ Q*_{11} \geq 0$

(ii) $2 - \lambda_2 + u_1 \geq 0;\ Q*_{12}\ (\ ) = 0;\ Q*_{12} \geq 0$

(iii) $3 - \lambda_1 + u_2 \geq 0;\ Q*_{21}\ (\ ) = 0;\ Q*_{21} \geq 0$

(iv) $6 - \lambda_2 + u_2 \geq 0;\ Q*_{22}\ (\ ) = 0;\ Q*_{22} \geq 0$

(v) $100 - Q_{11} - Q_{21} \geq 0;\quad \lambda*_1\ (\ ) = 0;\quad \lambda*_1 \geq 0$

(vi) $160 - Q_{12} - Q_{22} \geq 0;\quad \lambda*_2\ (\ ) = 0;\quad \lambda*_2 \geq 0$

(vii) $85 - Q_{11} - Q_{12} \geq 0;\quad u*_1\ (\ ) = 0;\quad u*_1 \leq 0$

(viii) $400 - Q_{21} - Q_{22} \geq 0;\quad u*_2\ (\ ) = 0;\quad u*_2 \leq 0$

price sets for the two regions are given in (4.11). The corresponding regional models are given in (4.12) and (4.13).

In all the $\partial\pi/\partial Q_{ij}$ in (4.12) and (4.13) unless $a_{ij} = 1$ the equilibrium in not well defined. If $a_{ij} > 1$ then negative per unit profits exist and no production occurs. If $a_{ij} < 1$ then positive profits exist and no level of Q^*_{ij} satisfies the FOC. Production will expand to the limit imposed by the availability of fixed resources in each region for the commodity with the highest per unit profits. When $a_{ij} = 1$ per unit profits are equal to zero and there is indifference between producing nothing and producing any other level up to the limit imposed by resource availability.

Identical regions, some resource use restrictions binding

In applied policy modeling the more common scenario would have some resource and some resource use flexibilities binding simultaneously (though never at the same time for the same resource unit). The results of that case are somewhere between the two extreme cases covered above. The inter-model results will again be inconsistent.

Non-identical regions and other inconsistencies

One problem frequently encountered when comparing or linking economic models is that they have different regional specifications. There are also potential problems arising when the regions from which the model data (technology and resources) are collected do not match model regions and weighted averages of the original data are formed. The problem is approached in two steps. First, the outcomes of a new national model with differing regional specification are compared to the original model. Secondly, the implications for the regional models given differing regional specifications is illustrated. It is assumed that the new regions for the national model, R^n_1 and R^n_2 are formed from the old regions by $R^n_1 = 1/3R^o_1 + 2/3R^o_2$ and $R^n_2 = 2/3R^o_1 + 1/3R^o_2$.

The resource constraints for the new regions are just the sums of the appropriate parts of the old regions, assuming the available data are sufficiently detailed to allow such a division. However, if new costs (technology) are formed using weighted portions of the old regional costs (the weights being based on resource proportions) the potential for large errors is introduced. The smallest error method for weighting the costs is that based on the likely solution (or some base) production pattern. An iterative solution and data reweighting scheme may be used to get this desired weighting. Even so, the results will be correct only as long as none of the parameters of the problem (such as demand levels or policy parameters) change.

The analytical form of the FOC from (4.2) are combined with some numerical values to give the FOC for this case as shown in (4.14). Note that it is assumed that all $a_{ij} = 1$ and that none of the resource use flexibilities are binding. The equilibrium production values are 85 for good 2 in region 1, 75 for good 2 in region 2, 100 for good 1 in region 2 and 0 for good 1 in region 1. With these numerical values (4.14) also shows exactly how the FOC are satisfied. It can be seen that in the solution resource 1 is used up. If one less unit of resource 1 were available the production of good 2 would have to shift to region 2 by 1 unit at a marginal cost increase of 4. Therefore the marginal value of resource 1 is 4 while the value of resource 2 is zero since some of the resource is unused. Since the last units of each good are produced in region 2, the marginal values are 3 for good 1 and 6 for good 2. The regional profit maximization model results for this case are analytically no different than for the previous case so are not shown.

The same analytical FOC forms as in (4.14) are used for the new regions as shown in (4.15) with the coefficients developed as explained above for the redefined regions. Now a different solution occurs but what it is cannot be seen. The marginal cost of production for each good is the same in both regions. Also region 1 now has sufficient resources to produce total demand for both goods. The national cost minimization problem is now indeterminate and the implied product prices are 3 and 4.13 for goods 1 and 2 regardless of which feasible production pattern is chosen.

The total cost of production is 960, the same as from (4.14). Now resources in both regions are valued at zero since in excess. Since the solution to (4.15) is degenerate it cannot be said to disaggregate to the solution from (4.14) using the weights (4.15) was formed with. Also note that with the new regions, if demands are increased the increased cost will be less than with the old regions. As stated by Hazell and Norton (1986, page 145): "aggregation bias is always in an upward direction; it overstates resource mobility by enabling farms to combine resources in proportions not available to them individually, and it carries the implicit assumption that all of the aggregated farms have equal access to the same technologies of production".

Equations (4.16) give the profit problem and solution for the old regions and (4.17) give it for the new regions. Neither (4.16) nor (4.17) duplicate the respective cost minimization solutions but (4.17) gives the larger error, particularly if the new profit maximization solution is compared to the old national model. The biggest problem, due to the constant costs, is the indeterminate solutions just as in the previous case. The crucial issue, given the differing regional definitions used in various models is how to force the solutions to the new profit

(4.15) (i) $3 - \lambda_1 + u_1 \geq 0;\ Q*_{11}\ (\ ) = 0;\ Q*_{11} \geq 0$

(ii) $4.13 - \lambda_2 + u_1 \geq 0;\ Q*_{12}\ (\ ) = 0;\ Q*_{12} \geq 0$

(iii) $3 - \lambda_1 + u_2 \geq 0;\ Q*_{21}\ (\ ) = 0;\ Q*_{21} \geq 0$

(iv) $4.13 - \lambda_2 + u_2 \geq 0;\ Q*_{22}\ (\ ) = 0;\ Q*_{22} \geq 0$

(v) $100 - Q_{11} - Q_{21} \geq 0;\ \lambda*_1\ (\ ) = 0;\ \lambda*_1 \geq 0$

(vi) $160 - Q_{12} - Q_{22} \geq 0;\ \lambda*_2\ (\ ) = 0;\ \lambda*_2 \geq 0$

(vii) $295 - Q_{11} - Q_{12} \geq 0;\ u*_1\ (\ ) = 0;\ u*_1 \leq 0$

(viii) $190 - Q_{21} - Q_{22} \geq 0;\ u*_2\ (\ ) = 0;\ u*_2 \leq 0$

(4.16) Max $\pi^o{}_1 = 3Q_{11} - 4Q_{11} - 4Q_{11} + 6Q_{12} - 4Q_{12} - 2Q_{12}$

FOC: $\pi^1{}_{o1} = 3 - 4 - 4 < 0$ so $Q^*{}_{11} = 0$

$\pi^1{}_{o2} = 6 - 4 - 2 = 0$ so $0 \leq Q^*{}_{12} \leq 85$, indeterminate

Max $\pi^o{}_2 = 3Q_{21} - 3Q_{11} + 6Q_{12} - 6\ Q_{12}$

FOC: $\pi^2{}_{o1} = 3-3 = 0$ so $0 \leq Q^*{}_{21} \leq 400$, indeterminate

$\pi^2{}_{o2} = 6-6 = 0$ so $0 \leq Q^*{}_{22} \leq 400$, indeterminate

(4.17) Max $\pi^n{}_1 = 3Q_{11} - 3Q_{11} - 4.13Q_{12} - 4.13Q_{12}$

FOC: $\pi^1{}_{o1} = 3 - 3 = 0$ so $0 \leq Q^*{}_{11} \leq 295$

$\pi^1{}_{o2} = 4.13 - 4.13 = 0$ so $0 \leq Q^*{}_{12} \leq 295$

Max $\pi^n{}_2 = 3Q_{21} - 3Q_{21} - 4.13Q_{22} - 4.13Q_{22}$

FOC: $\pi^2{}_{o1} = 3 - 3 = 0$ so $0 \leq Q^*{}_{21} \leq 190$

$\pi^2{}_{o2} = 4.13 - 4.13 = 0$ so $0 \leq Q^* \leq 190$

maximization problems of (4.17) to duplicate those of the old national shown in (4.14). Unfortunately, there seems to be no intuitive analytical method. Artificial constraints would have to be imposed. Even if non-linear forms were assumed and for a specified base solution inter-model consistency could be obtained the analyst will still be faced with the problem of adjustment of the aggregation weights for different policy scenarios (this is discussed in detail in the next section).

Transmittal of policy and other shocks

Before any model or system of models can be accepted for policy analysis its comparative statics must be verified. In the system implied in (4.2) with non-linear, well behaved technology and costs the comparative statics associated with shocks in exogenous parameters are straight forward and can be shown with the FOC or in the graphical presentation of Figure 1. However, just because the comparative statics of the aggregate model are consistent with the analyst's expectations, there is no guarantee about consistency with the micro-behavior of the individual firms.

It has already been show that non-linear well behaved aggregate functions are not consistent with aggregation of well behaved micro functions when linear aggregation rules are used. Use of non-linear aggregation rules result in indices of prices and quantities in the aggregate model which lack the desired interpretation. Hence, the comparative statics of the linear aggregation case must be considered.

Consider again the example of linear (homothetic) aggregators given in Chapter I. First, suppose that the homothetic assumption is true. The aggregators are linear with coefficients representing weights or shares. Is it reasonable for those shares to remain constant regardless of the policy scenario considered? If constant shares are not reasonable then can policy dependent share coefficients be derived or should an attempt be made to estimate what the shares might be under the new scenario. If it is reasonable for the shares to remain constant then employment of both aggregated and disaggregated models is not needed. The aggregate model could be solved and then disaggregated with the constant shares. If it is not reasonable for the shares to remain constant and enough information is available to specify endogenous shares or the appropriate shares after the policy impacts occur, then enough information is available that the analysis need not be performed; the impacts can be generated outside the model using the endogenous share information.

Suppose that the technology is not really homothetic but analysis proceeds as if it were. The issue then becomes an empirical one. First, shares must be chosen, presumably based on data from the status quo outcome. Secondly, caution must be used in interpreting the results. Regardless of the detail or structural elegance of the model policy shocks will tend to occur as linear deviations from some sort of an average. The estimated impacts will be either under or overstated with the direction and size of the error being an empirical question. With sufficient research budget the comparative statics of the model can be explored and compared to empirical data with iterative adjustments to coefficients until an acceptable solution is reached. It is also possible to include exogenous information in the form of model structural constraints in such a fashion as to guide the comparative statics.

Conclusions

The results of this chapter should not be taken to infer that aggregate models are not useful since they cannot be guaranteed consistent. It is probable that the demand for regional and national level natural resource policy analysis will continue to grow. It is also likely that researchers will be forced to become even more efficient in their work with less resources available for empirical testing and comparison of alternative structures.

The general implication is that more care in interpretation and application of the model results is needed. In particular, when model results are being communicated to non-modelers appropriate qualifiers as to the accuracy of the results and proper use should be given. The results shown in this chapter support the following conclusions.

A. Aggregation consistency theorems are available and can serve as useful guidelines for constructing aggregate models and for the type of qualifiers which need to be attached to aggregate results and to the corresponding welfare impact estimates.

1. Even though theorems for consistent aggregation are available, satisfaction of their requirements may not make sense in empirical economic analysis; the required restrictions on technology and preferences are generally not believable in an empirical sense.
2. For policy analysis, satisfaction of the stringent consistency theorems implies an absence of differential impacts across and within groups of agents and/or regions and so analysis is not so useful.
3. Since the satisfaction of the theorems are not so believable in an empirical sense, failure to satisfy strict consistency with chosen modeling structures should not be considered so serious.
4. The theorems do provide useful guidelines on the implications of choosing among alternative less-than-strict procedures when empirical models are being constructed.

B. The use of systems of models where the lower dimension models are components of the higher dimension model can be justified for several alternative scenarios of model structure and consistency. This is important in empirical national/regional and/or aggregate/firm level modeling systems for policy analysis.

1. Suppose that either consistent non-linear aggregation forms are used or that the issue of aggregation consistency is overlooked such that with the chosen models the first and second order conditions for optimization are satisfied. In this case intermodel pricing correspondences can be solved for so that the lower dimension models give the same primal and dual solution results for that entity and level of aggregation as are given by the higher dimension model.
2. If the chosen models involve linearity or are abstractions of reality to such an extent that artificial constraining methods are required to condition the primal solutions of the models in the system, information from the corresponding dual solutions can be used to adjust both the constraint levels and the pricing rules incorporated in the objective functions such that consistent solutions (and policy impacts) can be estimated.
3. In cases where model structure is sufficiently complex that analytical solutions are difficult to obtain and interpret some guidelines for intermodel correspondences can

be obtained from simpler models. Synthetic models of a simpler nature could be constructed and solved with numeric procedures to compare the effects of alternative model structures when analytical results are difficult to obtain.

4. These results imply that regional models can be used for natural resource policy analysis in the United States in a manner such that their solutions will be consistent with national analysis (which may not be performed). This means that models can be tailored to regionally specific issues while interregional externality issues are accounted for and that overall analysis will be more efficient.

CHAPTER V.

AN APPLICATION WITH EXISTING MODELS

Introduction and Outline of the Exercise

This chapter provides an application of the concepts from the previous chapters to a set of existing models. The example models in the previous chapters were necessarily simple so that results could be shown analytically. The models used in this chapter are far more complex and clear analytical correspondences are more difficult to find and illustrate. This exercise is designed to show why applied models are in general less rigorous from a theoretical standpoint than consistency theorems dictate. More important, the exercise also illustrates methods by which model consistency can be improved.

The nature of possible consistency links between two different existing models are explored in this chapter: a national cost minimization production and resource allocation model and a profit maximization model of the crop portion of Iowa agriculture. These models have been developed separately over time in response to differing policy analysis needs. Currently, commodity price and natural resource policy question have become intertwined and both models are being applied to the same problems.

The largest correspondence task of this chapter with the existing models is to explore the issue of existence of intermodel consistency links such as the national model constraint set shadow prices being interpreted as the regional profit model commodity program subsidies. In reality the chapter shows how primal solution consistency in the models comes at the expense of dual solution consistency and explains why this outcome will likely persist in similar applied models.

Various methods of artificial constraints placed in each model give consistent primal intermodel baseline results. Additional constraints also result in similar policy impact estimations for physical quantity variables from both models. However, the resource market implications coming from the models differ dramatically. This result indicates that the solutions are really driven at least partly by the artificial constraints rather than by underlying common or shared economic factors. This outcome implies that the policy estimates from both models may be in part a result of the artificial portions of the specification.

The national cost minimization model (NCM) assumes the existence of a Long Run Competitive Equilibrium (LRCE). However, policy induced and other distortions exist in the U.S. agricultural sector such that the "price equal to marginal cost at a minimum point on a long run average cost curve" condition of a LRCE is not satisfied; in this case assuming non-existence of distortions and evaluating the impacts of commodity and resource policy via comparative statics between alternative LRCE is not appropriate. Since a LRCE does not exist, the sector outcome is not one in which it looks as if profit maximizers are minimizing average cost. The prices on which decisions are based are not equal to marginal cost and positive profits do exist which in turn generate non-zero marginal rents in the resource markets. These non-zero marginal resource rents must be correctly accounted for in any welfare impact analysis.

The presence of non-zero marginal rents in resource markets require a modeling framework in which commodity price and resource policy links are made explicit. This model shortcoming is compensated for in the primal problem by historical crop acreage minimum constraints. When the NCM is forced to duplicate a baseline scenario via these artificial constraints, the solution dual resource values do not exhibit characteristics expected of an equilibrium situation. Likewise, the model imputed changes in resource rents due to a policy imposition are characteristic of a disequilibrium in which further pressures for adjustment exist.

The Iowa State Model (ISM) used in this study was developed to investigate the impact of commodity price and resource use policies on both commodity production and resource use outcomes.

The ISM takes prices as fixed, as if the state were one giant producer. The target price, deficiency payment, acreage set aside, loan rate, base acreage and CRP parameters of current farm policy are all included in the ISM. However, the participation choice is nearly fixed and what small freedom that does exist is in the form of a linear decision or "all or nothing" type of rule. More importantly, viewing one state in isolation from all other states does not account for pecuniary market and technological environmental externalities which readily move across state boundaries. This is particularly important when policy parameters are dependent on aggregate agricultural sector outcomes.

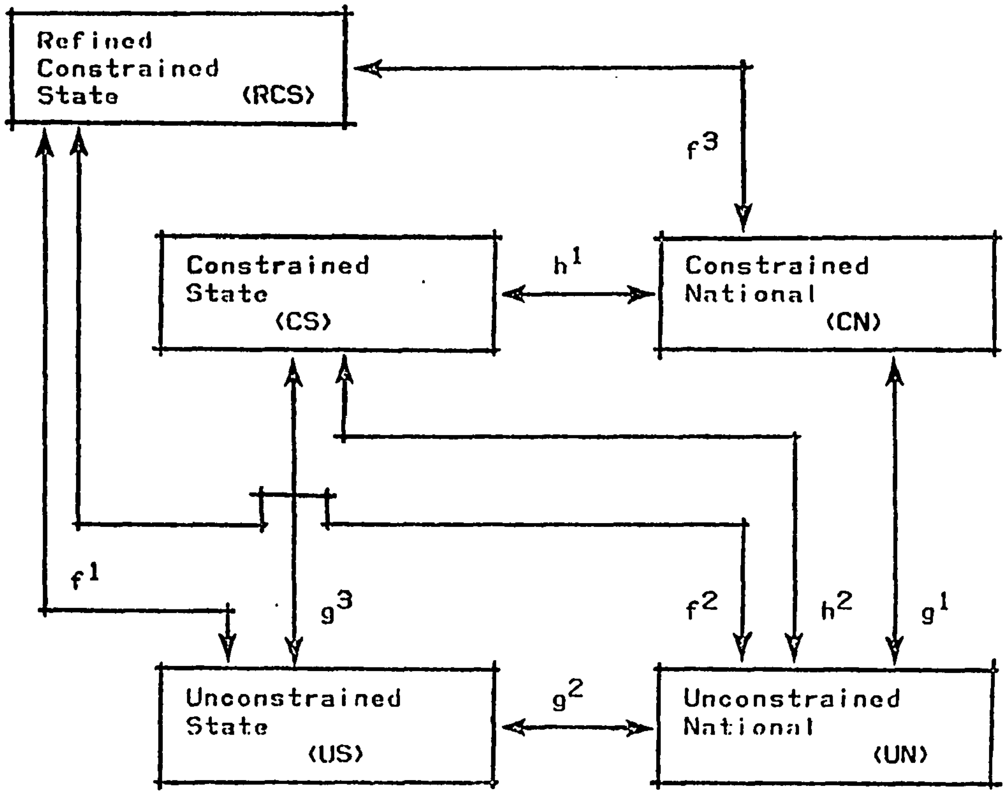
The commodity policy-profit incentives weakness of NCM and the partial equilibrium nature of ISM combined with the demands of current policy analysis have led to both models being used jointly. Consistency between the models is desired, not just for the primal quantity solution but also for the dual value solution which has implications for resource markets, welfare measurements, disequilibrium tendencies and so for actual resource allocation adjustments beyond the primal solution impacts.

The two models were taken as is and solutions obtained. For each model constrained solutions representing baselines were obtained. For each model the constraints on acreages or quantities required to get an adequate representation of some historical outcome for the baseline were then relaxed and free solutions obtained. With these four solutions the primal and dual aspects of the models are compared and a discussion of consistency problems is given. A refined constrained model is then formulated and improved possibilities for intermodel correspondences are examined to the extent possible.

An ideal exercise for determining better intermodel correspondences would involve modifying both models to obtain unconstrained versions and then an understandable constrained version from which consistent primal and dual solutions could be obtained. However, the state model is much more accessible in that both dual and primal solutions can be readily obtained while only summary tables exist for selected variables of the national primal and dual solutions. Also the state model is more easily modified and applied. The experiment will consist of using available constrained and unconstrained national model solutions while obtaining the same and other needed solutions for the state model with appropriate modifications.

Outline of the analytical exercise

The analytical exercise to be performed in this study is illustrated in Figure 5. Prior to the study constrained solutions were available for both models. These are labeled as Constrained State (CS) and Constrained National (CN). A nearly



Notation: CS, CN, and UN were available and US is developed.

g^1, g^2, and g^3 follow from chapter 4.

h^1 and h^2 are unknown but would be useful.

RCS is formed and f^1, f^2, and f^3 developed for use in place of h^1 and h^2.

Figure 5. Intermodel correspondences for applied example

unconstrained national solution (UN) existed and an unconstrained state model solution (US) was developed. If model regional and technological specifications are either assumed similar or else homogeneous such that aggregation imposes no biases several things could be learned about behavioral correspondences from these four models.

The relationship (g^1) between CN and UN and between CS and US (g^3) can be easily seen in both the primal and dual solutions. The change in the primal solutions will be exactly equal to the changes in the binding constraint levels. The change in the dual will be equal to per unit profit differences in the optimal and forced activities. Rent differences are due to profit differences multiplied by constraint levels. The price and quantity relationships (g^2) between UN and US can also be determined based on results of Chapter IV. However, as shown later in the section, correspondences between the constrained versions (CS and CN), h^1, cannot be isolated due to complicated model structure and ad hoc constraining methods. The h^2 correspondences between CS and UN are also not clear.

Given the modeling capacity constraints (requirement that linear methods be used) and the existing effects of in place agricultural policies, analysis of further government interventions need to be evaluated with constrained versions of the models. Isolation of resource market and welfare impacts of the policies when CS and CN are used require knowledge of h^1 and h^2, which is not derivable from the current constrained versions.

Based on evaluations made in finding an unconstrained state model (US) and the correspondences g^2 (between US and UN) and g^3 (between CS and US) a refined constrained state (RCS) model is developed. RCS is structured to have a primal solution at least as consistent as that coming from CS. From RCS the correspondences f^1, f^2, and f^3 can be more closely approximated than in the counterparts g^3, h^2, and h^1 of the original models.

The following sections of the chapter present structural detail and application implications, first for the state model and then for the national model. The aggregation paths implicit in generating Iowa results with the national model are also discussed. The remainder of the chapter presents results of all the models of Figure 5 except the refined constrained state model. It is shown how the primal solutions for the constrained models are similar, how the unconstrained models differ, and that the dual solutions are inconsistent. Values of the variables needed for the correspondences are presented and it is illustrated briefly why exact consistent links are difficult to determine. A section then presents a refined constrained state model and contrasts it to the previous cases. Finally, conclusions are presented.

The Iowa Profit Maximization Model

The Iowa model (IM) used in this study is part of the State Production Modeling System (SPMS) developed at the Center for Agricultural and Rural Development (CARD) at Iowa State University (Holtkamp, 1988). The SPMS was initially developed as the economic component of the Comprehensive Economic Pesticide Policy Evaluation System (CEPPES) (CEPPES, 1988). In conjunction with other models of CEPPES the IM has been used to evaluate the economic and environmental impacts of banning corn rootworm insecticides in the Corn Belt. Other applications of the IM include estimation of the impacts of expanding the CRP under various eligibility criterion, particularly for inclusion of waterway buffer strips.

The baseline version of IM is validated to simulate the historical 1986-87 costs, production and economics of agriculture in Iowa. The primary technology and cost data bases of the IM are built up from data prepared for use in the 1985 RCA Appraisal by CARD, the Soil Conservation Service and the Economic Research Service. Updating the 1985 RCA data to reflect 1986-87 conditions was accomplished using RUSE (Putman and Rosenberry, 1988). The model structure and data needed to supplement those used for the 1985 RCA so that a profit maximization model with commodity program policy parameters could be included was formulated at CARD from various sources (Holtkamp, 1988). The model description given below is more from a qualitative view, i.e., structure and implications, rather than on validity of empirical content.

The IM includes alternative production activities for barley, corn, legume and non-legume hay, oats, sorghum, soybeans and wheat. The entire state is modeled as if it were one large farm with nine separate fields, each comprised of an individual soil type. Management options include conventional and reduced tillage and substitution between chemical, hand and cultivated control of weeds. Also included is the ability to participate in commodity programs and a penalty for losing base acreage if not participating. The resource base to be used in production includes machinery and labor in addition to the land and base acreages. Profit maximization is assumed for the criterion function and risk implications are ignored.

The IM structure is rather detailed and complex and a lengthy discussion cannot be given in this paper. An attempt has been made to summarize the important features of IM in a schematic tableau representation (Table 1) without getting bogged down in detail. Lists of activities and rows indicating the rich detail of the model are given in Table 2 and Table 3. First the model rows and then the activities are discussed in detail below.

Table 1. Tableau for the Iowa state model

				Model Activities				
	RHS Value	RHS Type	# of rows	crops	comm. prog.	comm. prog. set aside	CRP set idle	CRP crop idle
	# of activities:			594	406	1	1	89
Row names								
Objective		N	1	−c	c	−c	−c	c
Land	NRI 1982[a]	L	10	a	0	0	0	1
Machinery	b[b]	L	45	a	0	a	a	0
Labor	b	L	9	a	0	a	a	0
Plantable base	b	L	45	0	1	0	0	0
Allow. base redu.	b	L	5	0	0	0	0	1
Historical base	Ag. Stats.[c]	L	5	0	1	0	0	0
Crop acres	Ag. Stats.	G	80	1	0	0	0	0
Tillage type	CTIC[d]	N	4	1	0	0	0	0
Bean Weed Trans.	0	G	1	1	0	0	0	0
Fertilizer use		N	3	a	0	0	0	0
Pesticide use	0	L	108	1	0	0	0	0
Program partic.	0	G	45	1	−1.2	0	0	0
Prog. set aside	0	L	1	0	0.2	−1	0	0
Prog. part. acres	0	G	5	0	1	0	0	0
CRP set aside	0	L	1	0	0	0	−1	1
Crop transfer	0	G	8	a	0	0	0	0
Alloc. crop mix	0	E	8	a	0	0	0	0
CRP land type mix	0	E	10	0	0	0	0	a
CRP enrollment	0	G	2	0	0	0	0	1
Crop tillage type		N	23	1	0	0	0	0
Max. land idle	50%	L	9	0	0.2	0	0	1
		Total=428						

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[a] NRI 1982 is the 1982 Natural Resources Inventory.

[b] A "b" indicates this value determined by a fixed demand prior run.

[c] Ag. Stats. refers to various agicultural statistics.

[d] CTIC is the Conservation Tillage Information Center.

Model Activities

CRP land mix	bean weed tran	crop type mix	buy chem.	hire labr	subs labr	sell crops
1	1	2	90	3	5	8
0	-c	0	-c	-c	0	c
0	0	0	0	0	0	0
0	a	0	0	0	0	0
0	a	0	0	-1	a	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	-1	0	0	0	0
0	0	0	0	0	0	0
0	-1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	-1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	-1
0	0	-a	0	0	0	0
a	0	0	0	0	0	0
1	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Iowa model rows

Resources   The resources specified in IM include land, machinery, labor, base acreages and plantable base acreages. Each of these is discussed separately in more detail.

The cropland base of Iowa is divided into nine groupings of USDA Land Use Capability Classes and subclasses. Since these groups are nearly equal to those of the national model the definitions are shown in Table 4 so that contrasts can be seen. This grouping is a modification of that used in the 1985 RCA to better address water quality issues. The individual land groups are each modeled as a "less-than" row in IM with RHSs determined from RUSE. A land use total row is non-constraining and simply adds up the land use across groups. Production activities in IM are land group specific but some land idling or set aside activities require a composite of the land groups, usually based on historical observations.

Enterprise budgets of the Firm Enterprise Data System (Kletke, 1979) format comprise the cost and technology data for IM. However, individual machine requirements are classed into 15 groups and availability is specified by season: spring, summer and fall. The RHSs of these constraints do not represent actual data. A pre-baseline run of the model was made with no machine use limitations but with a requirement that the 1986-87 levels of

Table 2. Definition of activities in state model

CA	Hay and Small Grains Allocation	1-2[a]	'CA'
		3-5	(SMG or HAY)
CR	Conservation Reserve Enrollment	1-2	'CR'
		3-5	Crop (BAR, CRN, OTS, SRG, WHT)
		6	'C'(current) and 'F'(future)
		7	Land group (1-9)
CRPCT	CRP land group allocation	1-6	'CRPCT'
BNPV	Base acreage idle penalty	1-2	'BNPV'
		3-5	Crop (BAR, CRN, WHT)
		6	Land group (1-9)
BW	Bean Weeding	1-2	'BW'
D	Dryland cropping	1	'D'
		2	Land group (1-9)
		3-4	Rotation Code (20 available)
		5	Tillage class A = Fall plow, conventional B = Spring plow, conventional C = Conservation tillage D = Zero tillage
		6	Scenario number
Q	Labor hire	1	'Q'
		2-3	Season (SP, SU, FA)
R	Program participation	1	'R'
		2	Land group (1-9)
		3	Set aside land group (1-9)
		4-6	Crop (BAR, CRN, OTS, SRG, WHT)

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[a] Numbers refer to column name space in LP MPSX format.

TABLE 2. Continued.

SB	Substitute labor	1-2	'SB'
		3-4	Season (SP, SU, FA)
		5-6	Labor type (Hired or Operator)
S	Set aside (and green cover, program acres)	1	'S'
SCRP	Set aside (green cover on CRP acres)	1-4	'SCRP'
X	Buy chemicals	1	'X'
		2-4	Crop (CRN, SOY)
		5	Pesticide 1 = alachlor 2 = atrazine 3 = dual 4 = sencor 5 = treflan 6 = terbufos
		6	Land group (1-9)
Z	Crop sell	1	'Z'
		2-4	Crop (BAR, CRN, LHA, NLA, OTS, SRG, SOY, WHT)

actual commodity production occur. The optimal pre-baseline machinery use or requirements were then increased by five percent to become machinery resources in the baseline model. The methodology clearly limits the range of impacts available from policy estimations with the model; data on actual machine availability were not sufficient for model development.

Table 3. Definition of rows in the state model

Resource rows

LDY	Land availability	1-3[a]	'LDY'
		4-5	Land group (1-9)
M	Machinery availability	1	'M'
		2-3	Machinery code (1-15) 1 = Primary tillage 2 = Secondary tillage 3 = Fertilizer and chemicals 4 = Cultivating conventional 5 = Harvesting grain 6 = Haying 7 = Power units 8 = Other 9 = Conventional planting (CRN, SOY) 10 = Conventional planting (small grains) 11 = Minimum till planting (CRN, SOY) 12 = Minimum till planting (small grains) 13 = No till planting (CRN, SOY) 14 = No till planting (small grains) 15 = Seeders (for set aside land)
		3-4	Season (SP, SU, FA)
N	Labor constraint	1	'N'
		2-3	Season (SP, SU, FA)
		4-5	Type (Hired, Operator, or Substitute)
NPV	Planted base acreages	1-3	'NPV'
		4-6	Crop (BAR, CRN, OTS, SRG, WHT)
		7	'T'
		8	Land group (1-9)
O	Historical crop bases	1	'O'
		2-4	Program crop (BAR, CRN, OTS, SRG, WHT)

---

[a] Numbers refer to column name space in LP MPSX format.

TABLE 3. Continued.

Accounting Rows			
L	Individual crop acreage	1	'L'
		2-4	Crop (one row for each of eight crops)
		5	Land group (1-9)
YTOT	Tillage type	1-4	'YTOT'
		5	tillage A= Fall plow, conventional B= Spring plow, conventional C= Conservation tillage D= No till
Transfer Rows			
BWT	Bean weeding	1-3	'BWT'
F	Fertilizer buy and use	1	'F'
		2-3	Fert. (NI, PO, and PH)
		4-5	'BY'
G	Program participation	1	'G'
		2	Land group (1-9)
		3-5	Crop (BAR, CRN, OTS, SRG, WHT)
I	Program land set aside	1	'I'
ICRP	CRP land set aside (enrollment)	1-4	'ICRP'
K	Crop transfer	1	'K'
		2-4	Crop (one row for each of eight crops)
P	Pesticide use	1	'P'
		2-4	Crop (CRN, SOY)
		5	Pesticide 1 = alachlor 2 = atrazine 3 = dual 4 = sencor 5 = treflan 6 = terbufos
		6	Land group (1-9)
R	Program participation Acres	1	'R'
		2-4	Crop (BAR, CRN, OTS, SRG, WHT)
		5-7	'TOT'

TABLE 3. Continued.

Miscellaneous fixity constraints

ALLOC	Hay and small grains allocation	1-5	'ALLOC'
		6-8	Crop (one row for each of eight crops)
CRLDY	CRP Land group allocation	1-5	'CRLDY'
		6-7	Land group (1-9)
CRPTOTC	Current CRP enrollment	1-7	'CRPTOTC'
CRPTOTF	Future CRP enrollment	1-7	'CRPTOTF'
L	Crop acreages	1	'L'
		2-4	Crop (one row for each of eight crops)
		5-7	'TOT'
Y	Allowable tillage by type and crop	1	'Y'
		2	tillage A= Fall plow, conventional B= Spring plow, conventional C= Conservation tillage D= No till
		3-5	Crop (one row for each of eight crops)
W	Allowable total land setaside	1	'W'
		2	Land group (1-9)

Labor resources are specified by season (spring, summer and fall) and by operator and hired. Hired labor is assumed available at a fixed cost in unlimited quantities but the ability to substitute hired labor for operator labor is limited. Levels of operator labor available were developed from secondary Iowa data sources.

Table 4. Definition of land resources in Iowa model (CEPPES, 1988)

Model land group	USDA Land Capability Class/subclass[a]
1	I
2	II_e
3	III_e
4	IV_e
5	II_c, III_c, IV_c
6	II_s, III_s, IV_s
7	II_w, III_w, IV_w
8	V_e, VI_e, VII_e, $VIII_e$
	V_s, VI_s, VII_s, $VIII_s$
9	V_c, VI_c, VII_c, $VIII_c$
	V_w, VI_w, VII_w, $VIII_w$

[a] Subscripts indicate nature of limitation to normal use: e for erosive, c for climate, s for shallow/stony, and w for wet.

Commodity program base acreages are specified for barley, corn, oats, sorghum and wheat using actual data. These are reduced to account for the base retirement requirements associated with CRP enrollments, resulting in the planted base acreage resources. The base reduction requirement is conditioned on proportions of individual crop acreage as base acreage and for future CRP signups assumes that past trends continue. When the model was first encountered in this study the planted base acreage rows were equalities, requiring that either the "after CRP" base be used in program participation or incur a penalty equal to the discounted Net Present Value (NPV) of lost base earnings. That equality requirement had an adverse effect on the dual solution where very large positive shadow prices were offsetting very large negative shadow prices in other places. These constraints were changed to "less-than" and the dual solution became realistic with hardly any effect in the primal solution.

Accounting rows Accounting rows are those with unconstrained RHSs which simply total up levels of primal variables. The constitute only a small portion of the IM (84 of 428 rows). They IM has crop acreage rows for eight crops on each of nine land groups and an additional total over land groups row for each crop; these rows can be constrained to upper or lower limits or left free. Four types of tillage are tracked in the model solution. At one time RHS values existed for these rows but the machinery availability constraint effectively screens tillage type to realistic values. These rows have no impact on either the primal or the dual solution.

Transfer rows Transfer rows are those with zero right hand sides which link solution values of different activities. They constitute a large portion of IM. Input use or cost transfer rows linking production activities with buying or renting activities include bean weeding (BWT), fertilizer purchase and use (F), program participation by crop and land group (G), land idling for set aside cover (I and ICRP), crop commodity transfer (K), pesticide use (P) and program participation (R). Since these rows either are specified as equalities with a RHS of zero or incur a solution value RHS of zero by default they have little impact on the dual solution rent evaluation. However, they often drive the primal solution and may have very large shadow prices. Since the RHSs are zero welfare implications are not forthcoming here.

Miscellaneous fixity constraints Miscellaneous fixity constraints generally enter into model structure as a means of forcing the primal solution (as in the linear technology case of the regional models in the previous chapter). However, these constraints usually have drastic impacts on the dual solution beyond the small primal corrections they force. The constraints also severely limit the ability of the model to respond to changes, and so limit the ability to use the model for policy analysis. In practicality, the IM, like other models, is only a rough approximation of the real economy and requires these type of constraints to overcome weaknesses.

It was found that the IM would specialize entirely in either legume or non-legume hay and entirely in only one small grain, depending on relative profitability. Apparently, there are unmodeled factors leading farmers to continue to produce a diversity of these crops despite the profit differences. Therefore, crop mix allocation activities were specified for hay and for small grains (ALLOC). For hay ALLOC requires that the acreage of legume hay relative to non-legume hay will always be the same as in some base period. For the small grains it means that the proportions of total small grains devoted to each component are constant. The results of earlier chapters of this paper indicate that such a requirement implies the existence of total hay and small grain production functions which are homothetic in their individual crop components. This seems to limit the flexibility of the model quite severely, particularly when combined with the pre-baseline solution method of deriving the baseline and policy scenario machinery constraints.

The profit maximization nature of the IM combined with the assumed purely homogeneous land groups would result in only the poorest land being enrolled in the CRP. In reality, portions of various land groups have been enrolled. To overcome this model weakness, constraints cause that the portions of total CRP by land group for both current and future signups to duplicate the historical proportions (CRPTOTC). In addition, the current and future CRP signup quantities are specified as equalities (CRPTOTC and CRPTOTF). The shadow prices on these constraints can be added to the assumed CRP rent in the activity objective coefficient to see the implied rent needed to voluntarily achieve the required signup level.

The IM initially contained accounting rows for the total acreage of each crop. Since the summary tables for Iowa from the national model indicated a slightly different acreage allocation (particularly with minor acreage of small grains) these accounting rows were changed to minimum acreage constraints with RHS levels as nearly as possible equal to the national model.

The IM has the capability of restricting tillage type acreage for each crop (Y). Currently these constraints are turned off since the method of deriving the machinery constraints takes care of the problem of realistic tillage type adoption rates.

Finally, to prevent model specialization beyond what is realistic in land idling given the heterogeneous nature of Iowa land vs the homogeneous land of the IM, allowable land group set aside maximums are specified. These are set at 50 percent of the resource acreage in this study.

Iowa model activities

Cropping As shown in Tables 2 and 3 dryland cropping activities are defined for several rotations and management practice combinations for each land group. These activities include variable cost except for pesticides and some weeding requirements for soybeans and corn and use land, machinery and labor resources. The activities generate individual crop acres, acres to which program participation can be applied and commodity production. The activities also require weeding, fertilizer and pesticides all of which are supplied by other activities via transfer rows (all crops except corn and soybeans have a fixed level of these costs in the objective function coefficient and there is no choice).

Commodity program participation Commodity program participation activities require planted crop acres, base acreage and plantable base acreage resources and generate revenue and set aside acres. For barley, corn, oats, sorghum and wheat the deficiency and diversion payments are applied to base yields to determine profits. For soybeans the state wide participation rate is assumed to apply equally to all acres and the loan rate is factored into the commodity price in the selling activities. These activities are land group specific in two ways. First, they are by land group on which the participating crop is produced. Secondly, they are by land group which is used to meet the set aside requirement. Any land group can be used to meet the set aside requirement for production and participation of any other land group.

Land set aside Separate activities are specified for setting aside land for program participation and as part of the CRP enrollment. The function of these activities is to establish and maintain the required green cover crop on the land. The annualized cost of the cover establishment and the annual maintenance cost are included in the objective function coefficient. These activities also require machinery and labor.

CRP enrollment and land group allocation Individual activities for both current and future CRP enrollment are specified by land group in the IM. A CRP land group allocation activity forces the land groups to be enrolled in the same proportion as actual enrollments in some base period. The CRP enrollment activities are also differentiated by crop and for each acre of enrollment a proportion of an acre of the crop base is used up; the proportion is that of base acreage to total acreage of the crop.

Crop type allocation The IM contains one activity for hay and one activity for small grains to require components of these crop types to be produced in constant proportions to each other. These activities interact with the crop acreage and the ALLOC rows.

Crop selling Crop commodity selling activities are in the model for each of the eight crops. The objective function coefficients of these activities are the market prices of the crops.

Miscellaneous activities Bean weeding activities are specified in the model on a per acre basis. These represent a hand weeding of the soybeans. The IM can be altered to allow substitution between pesticides, extra cultivation and hand weeding. Buy chemical activities are in the model by crop, land group and chemical. The objective function coefficient is the per acre cost. Additional labor can be hired and is substituted for operator labor in some applications up to specified limits by the Substitute labor activities.

The National Model

The Agricultural Resource Interregional Modelling System (ARIMS) was originally developed at the Center for Agricultural and Rural Development (CARD), Iowa State University, for use in the Second Resource Conservation Act (RCA) Appraisal (English et al., 1988; Robertson et al., 1987). ARIMS has subsequently been modified and updated for use in analysis of current policy issues (AAEA, 1988). Examples include expanding the CRP under alternative land eligibility criterion, alternative levels of Conservation Compliance erosion allowances and the impact of a five cent tax on nitrogen fertilizer use.

ARIMS consists of a large scale national linear programming (LP) model and several supporting data sets and models. This set of models simulates economic activity in and between seven sectors of U.S. agriculture: crop production, livestock production, pasture/range production, irrigation requirements and costs, land availability, final and intermediate commodity demand and transportation.

ARIMS utilizes three different regional definitions. The first and primary set of regions consist of 105 producing areas (PA) (Figure 6). These areas are the basic regions of crop production. The land availability and irrigation sectors are defined at this level. These areas are the sub aggregates of water basin areas used in watershed analysis during the 1970s. They were chosen for delineating crop production because of the irrigation sector data availability and the inability to convert that data to other regions.

The second set of regions serve jointly as the 31 market regions (MR) and 31 livestock producing areas (Figure 7). At this regional level commodity demands, nitrogen purchase, livestock feeding and production and transportation hubs are defined. Transportation routes between MRs for both crops and livestock are defined as truck, train, and barge where appropriate. Three of the market regions serve as commodity export links to the rest of the world.

The final set of regions are the ecosystems. Pasture and range coefficients are defined at this level. The ecosystems cannot be mapped as clearly as can the PAs and MRs since they are based on natural vegetation characteristics rather than on location (USDA, 1988). A given PA may contain acres from any or all 34 ecosystems. And, a given ecosystem may be represented in all the PAs.

In addition to the regions explicitly contained in the model structure, coefficient development involved data sets by county, state, USDA Farm Production Region and Major Land Resource Areas (MLRA) (USDA, 1981) regions. Data from these regions were adapted to the model regions based on 1982 Natural Resource Inventory (82 NRI) (USDA, 1984) crop acres. The calibration of the model using input data minimized the overstatement of productive ability and understatement of costs otherwise associated with area aggregations (Hazell and Norton 1986). The solution summary tables for ARIMS, such as those for Iowa used in this study, also rely on weighted aggregation according to fixed regional shares. This can result in apparent inconsistencies in model summary lists of crops grown in the state and so forth.

The equations comprising ARIMS have a coefficient matrix structure which is nearly block diagonal. At the national level, there are cotton demand constraints, national land total and various feed use correction constraints. Only transportation links exist between MRs and PAs. There are only a few constraints (other than demand) that link PAs within each MR.

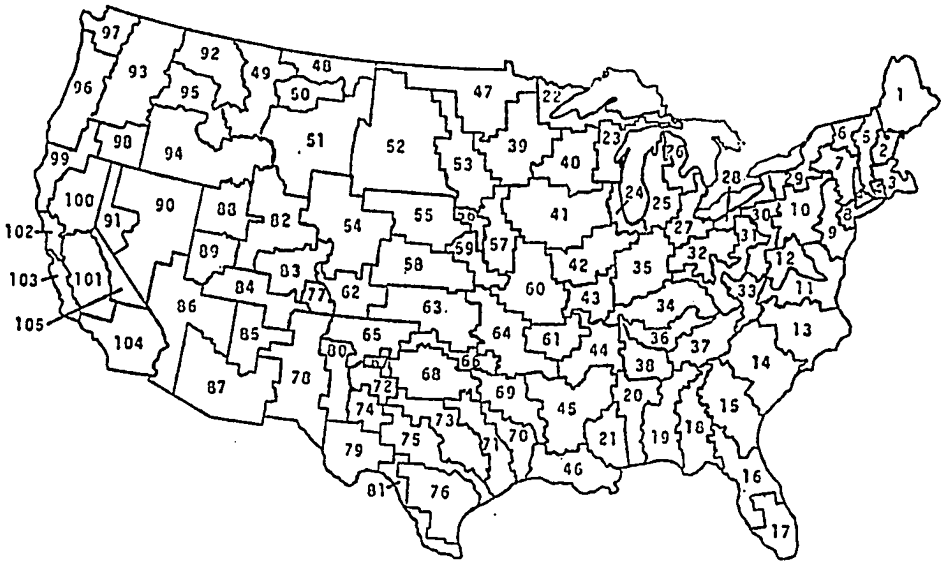

Figure 6. The crop producing areas

Figure 7. The livestock producing and market regions

The justification for the national least cost minimization criterion is based on the initial use of ARIMS to examine alternatives for an assumed long run competitive equilibrium (Robertson et al., 1987). In a long run competitive equilibrium producers each minimize long run average costs (Silberberg, 1974) and only the most efficient remain in production (Layard and Walters, 1978). Constraints representing fixed demands are placed in the model and their shadow prices in the final solution become imputed commodity prices, under the assumption of marginal costs equals price in the competitive equilibrium. However, various factors not modeled may influence production decisions and ARIMS contains production shift constraints to adapt the model to better simulate real world behavior as influenced by these unspecified factors. References on specifying this type of artificial structure are available but an analysis such as performed in this study has not been done (Henderson, 1959; McCarl and Apland, 1986; Miller, 1972).

Crop production

Crop production is by activities representing a one-to-six year rotation sequence and a combination of a tillage method and conservation practice on a specified land group (see Table 6). The activities include barley, corn (grain and silage), cotton, hay (legume and non-legume), oats, peanuts, sorghum (grain and silage), soybeans, summerfallow, sunflowers and wheat and in some cases double crops (see Table 5). The water and fertilizer needs of other exogenous crops are also accounted for.

Tillage practices include conventional (with residue over winter and without), conservation and zero tillage. Conservation practices which can replace straight row are contouring, strip cropping and terracing. One tillage practice can be combined with one conservation practice on land groups where appropriate (see Table 6). In addition, in some PAs strip cropping is defined for wind rather than water erosion control. No-till is any practice leaving more than 85 percent residue cover on the ground at the time of planting. Conservation till leaves between 30 and 85 percent residue cover. Conventional tillage is differentiated as having the primary operation in the fall or spring.

Livestock production

Livestock production includes beef (both grain and roughage fed), pork and dairy (milk) (see Table 6). These production activities are specified by market region and within each MR are differentiated by size. Alternative feeding activities are defined for the different feeds produced and purchased in the model for each class of livestock. Livestock production results in manure which enters the transfer rows as fertilizer the same as commercial purchases of fertilizers.

Table 5. Commodities produced in the National model[a]

Crop Code	Commodity definition
1	Barley
2	Corn grain
3	Corn silage
4	Cotton
5	Legume hay
6	Non-legume hay
7	Oats
8	Pasture
9	Peanuts
10	Sorghum grain
11	Sorhum silage
12	Soybeans
13	Summer fallow
14	Sunflowers
15	Spring wheat
16	Winter wheat
17	Establish legume hay
18	Establish non-legume hay
81	Winter wheat-soybeans DC
82	Non-legume hay-winter wheat DC
83	Sorghum-winter wheat DC
84	Corn-sunflower DC
85	Corn-sorghum DC
86	Sorghum-soybeans DC
87	Corn-soybeans DC
88	Wheat-peanuts DC
89	Sorghum-oats DC
90	Oats-soybeans DC
91	Oats-peanuts DC
92	Barley-soybeans DC
93	Barley-corn silage DC
94	Winter wheat-corn silage DC
95	Barley-sorghum DC
96	Barley-corn DC
97	Oats-corn silage DC
	Beef[b] grain fed
	roughage fed
	Milk
	Pork

---

[a] In the crop definition, a DC indicates a double crop sequence grown within one year.

[b] Other livestock units are produced but they serve as intermediate inputs for these final goods.

Table 6. Definition of land resources and allowable conservation practices in the National model[a]

Land Group[b]	USDA Land Capability Class/Subclass[c]	Straight Row	Contour	Strip Cropping	Terracing
1	I, IIwa, IIIwa	X			
2	IIe	X	X	X	X
3	IIe	X	X	X	X
4	IVe	X		X	X
5	IIc, IIIc, IVc	X		X	
6	IIs, IIIs, IVs	X	X[d]	X	
7	IIw, IIIw, IVw	X			
8	V, VI, VII, VIII	X	X	X	X

---

[a] Each "X" incorporates fall and spring conventional, conservation and zero tillage practices in combination with the conservation practice.

No till = more than 85 percent residue cover after planting.
Conservation till = between 30 and 85 percent residue after planting.
Conventional till = less than 30 percent residue cover after planting (without winter cover = primary tillage in spring).

[b] These are the land groupings defined for the 1985 Resource Conservation Act Appraisal.

[c] The subclass subscripts are the same as in Table 4 except that wa indicates that the wetness problem has been adequately treated.

[d] This practice is not allowed on sand.

Intermediate livestock activities exist for producing offspring, etc., which can be used between the different types of producers, i.e., farrow to finish or feeder pigs only producers. The sector must also account for the nutritional needs of all exogenous (to the model) livestock production in the U.S.

Land and water resources

The land base is determined from the 1982 Natural Resources Inventory (82NRI) compiled by the Soil Conservation Service (USDA, 1984). All land which was found to be currently, or that had a recent history of cropping, was counted as cropland. Non-cropland was characterized with regard to potential for conversion to crops and activities for converting high and medium potential land were defined.

Land requirements for urban and other exogenous crops have been taken out of the land base given projections from other models. The water requirements of exogenous crops and for all exogenous livestock (all U.S. except those specifically listed as being included in the model) are also removed from the resource base.

CRP enrollments, both current and future are determined exogenously prior to model application and then the acreages subtracted from the model land resources. These exogenous determinations are by PA and land group so ARIMS is only free to choose whether to enroll dry, surface or ground irrigated land to meet the CRP enrollment constraint for each PA and land group.

Land constraints are specified as equalities, forcing all the crop land to either be cropped, enrolled in CRP or idled with a green cover crop. As shown earlier in this chapter this requirement really distorts the dual solution and the associated welfare implications.

Miscellaneous fixity constraints

ARIMS contains various fixity constraints which limit adjustment in the agricultural sector. These constraints are based on the observation that if unconstrained the model tends to adjust more quickly to even small parameter changes than do actual producers. Such constraints are common in programming models since they are only simplified representations of the complex real world. Examples in the literature include (Henderson, 1959; McCarl and Apland, 1986; Nugent, 1970; and Sahi and Craddock, 1974).

Constraints exist in ARIMS which require a predetermined amount of irrigated land by water source type at the producing area level and in total at the national level. Without these ARIMS does not recognize that the fixed cost incurred in establishing the original irrigated acreage base likely keeps production occurring to service outstanding debt, etc. These constraints impact both the primal and dual solutions of the model; the distortions are particularly large in the dual solution shadow prices.

Adoption (or abandonment) of conservation practices involves the same sort of cost/debt retiring complications as the irrigation investment. Terracing and contouring also produce water movement channelling benefits not recognized at all by ARIMS. Therefore ARIMS is also artificially constrained with regard to these variables.

ARIMS requires that terraced acres by producing area be maintained at their 1982 levels. Adoption of conservation and zero tillage is limited to a percentage change from the past acreages (generally set at 120 percent of the 1986-87 levels). The fixed costs of terracing are not in the model; however, the shadow prices associated with the minimum acreage constraints are more than the variable production cost differences between production activities utilizing terracing and the available alternative methods.

The largest set of constraints in ARIMS and the ones which this study is most oriented towards are those requiring 80 percent of 1986-87 individual crop acreages by market region. These are completely ad hoc although the authors cited above in this section give various justifications and econometric methods for estimating the percentage change flexibility parameters. Since the unconstrained model represents a long run competitive equilibrium these constraints can be interpreted as representing current distortions from that equilibrium. In this interpretation the shadow prices on these constraints divided by yield can be correlated with the commodity subsidies in a related profit maximization model. This comparison is only strictly valid when the market prices of the profit maximization model are the equilibrium marginal costs in the national model as shown in the theoretical models of Chapter IV. With the complex structure and differing baselines of the models evaluated here such simple correlations are not derivable.

Baseline assumptions

Production costs are representative of 1980, while yields, acreage constraints and overall demand estimates have been updated to predicted 1990 levels. A highly simplified tableau representation of a portion of the model showing a typical PA and MR activities and row interactions is given in Table 7. The model is sufficiently complex that in other sources cited above several pages are devoted to example tableaus.

Table 7. Simplified national model tableau representation

			Model activities					
			at the PA level					
	RHS	RHS	cropping activity			idle w/green cover		
Activities	source	type	dry	surf.	ground	dry	surf.	ground
Objective		N	c	c	c	c	c	c
Land: dry	NRI82[a]	L	1	0	0	1	0	0
surf.	NRI82	L	0	1	0	0	1	0
ground	NRI82	L	0	0	1	0	0	1
Water: surf.	Spec.[b]	L	0	1	0	0	0	0
ground	Spec.	L	0	0	1	0	0	0
Crop demand	FAPRI[c]	G	a	a	a	0	0	0
Fert. transf.	0	L	a	a	a	0	0	0
Erosion		N	a	a	a	a	a	a
Tillage max.	CTIC[d]	L	1	1	1	0	0	0
Terrace min.	NRI82	G	1	1	1	0	0	0
Crop min.		G	a	a	a	0	0	0
Irrigat. min.	Spec.	G	0	1	1	0	0	0
Meat demand	FAPRI	G	0	0	0	0	0	0
Grass transf.	0	G	0	0	0	0	0	0
Pasture max.	Spec.	L	0	0	0	0	0	0
Ecosyst. res.	NRI82	L	0	0	0	0	0	0

[a] NRI82 is the 1982 Natural Resources Inventory.

[b] Spec. indicates that several specialized data sets were used.

[c] FAPRI is the national demand levels which are apportioned to regions using the NIRAP fixed weights.

[d] CTIC is the Conservation Tillage Information Center.

	at the MR level				national transportation				Eco-syst. grass
land	livestock feeding			fert.	crops		livestock		
conv.	grain	rough.	grass	buy	MRi-j	MRj-i	MRi-j	MRj-i	prod.
c	c	c	c	c	c	c	c	c	c
-1	0	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	-a	-a	0	0	-a	a	0	0	0
0	-a	-a	-a	-a	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	a	a	a	0	0	0	-a	a	0
0	0	0	-a	0	0	0	0	0	a
0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	1

Result Comparisons from Original Models

Intermodel consistency in the primal solutions

In the constrained (baseline) versions both the national and the Iowa state models were constrained to simulate the acreage and commodity production outcomes historically seen in 1985-86. Table 8 compares the resource definitions and availability by model (resource definitions and model structure are explained more clearly in later chapters). The only resource specified in the national model (ARIMS) is land, though land is divided into dry and irrigated by water source which is not done in the state model. The state model contains resource constraints for 15 classes of machines and operator labor by three seasons and commodity program base acreages in addition to land. The machinery constraints are actually 105 percent of the machinery required to produce the 1985-86 commodity levels in a pre-baseline specification of the model where the machinery constraints were completely relaxed.

Commodity production totals from the primal solutions are compared in Table 9, both for the original constrained (historical level) models and versions without crop acreage constraints. In the constrained case the crop acreages across models are fairly consistent though it should be noted that legumes, hay, oats, sorghum and wheat are near their upper feasible limits in the state model (but still below the ARIMS).

Table 8. Resource definition and availability by model

			Iowa State Model		
		ARIMS	Total	Spring	Summer
Fall					
Land (1000s acres):	1	6135	2898		
	2	6197	5887		
	3	6287	6826		
	4	1456	1980		
	5	2	0		
	6	876	783		
	7	4988	7650		
	8	687	786		
	9	0	69		
	Total	16627	26881		
Machines (1000s hours):					
1=primary tillage			0	0	4908
2=secondary tillage			6101	0	101
3=fert. and chemicals			6869	1338	749
4=cultivating, convent.			0	11537	0
5=harvesting			0	740	4820
6=haying			2549	5063	2055
7=powers			19633	18055	10686
8=other			free	free	free
9=convent. plant(crn,soy)			1963	0	0
10=convent. plant(sml grn)			75	0	20
11=min. til plant(crn,soy)			1848	0	0
12=min. til plant(sml grn)			0	0	578
13=no till plant(crn,soy)			104	0	0
14=no till plant(sml grn)			2	0	0
15=seeders (set aside plnt)			0	0	0
Operator labor (1000s hours)			25037	42875	28749
Base acreages (1000s acres)					
Barley			0.2		
Corn			13632.0		
Oats			28.0		
Sorghum			12.0		
Wheat			55.0		

With the individual crop acreage constraints deleted the free Iowa model behaves as predicted for linear technology in Chapter IV, i.e., specialization in the crops yielding the highest profit margin (profit margins are shown in next section). Note that not only is corn acreage up but yields also increase, indicating a shift of better land to corn production. Soybean acreage and production increase more than they did for corn but yields decrease.

The unconstrained national model also has significant changes but does not specialize to the extent of state model. Only the individual crop acreage and maximum conservation and zero tillage constraints were removed; constraints on total land use, minimum irrigation and terracing remained binding. Soybean acreage and production increased while corn decreased as did all other crops.

The relatively stable solution for Iowa indicates a maintenance of a competitive comparative advantage for the state over other states despite national distortions due to subsidies. The bias towards corn production due to current (and recent historical) corn subsidies and acreage base requirements can also be seen. In a subsidy distortion free scenario other crops, particularly soybeans, would gain in acreage at the expense of corn.

Table 9. Primal solution characteristics--commodity production

	Barley	Corn	L-hay	NLhay	Oats	Srgum	Soybn	Wheat
Acres(1000s)								
State Con.[a]	4	9819	1200	643	1000	670	8170	108
State Free[b]	0	10125	0	0	0	0	11490	0
Nat. Con.[c]	71	9374	1199	77	1265	1334	7433	399
Nat. Free[d]	66	7527	1136	0	820	1932	9314	0
Output(millions)								
State Con.	0.3	1330.0	4.4	1.1	52.5	60.6	266.2	4.2
State Free	0.0	1432.3	0.0	0.0	0.0	0.0	364.2	0.0
Nat. Con.	3.9	1137.4	4.7	0.1	66.2	112.5	354.2	17.2
Nat. Free.	3.6	891.0	4.8	0.0	40.1	164.6	442.2	0.0
Yields								
State Con.	65.1	135.1	3.7	1.8	52.8	90.4	32.6	39.2
State Free	0.0	141.5	0.0	0.0	0.0	0.0	31.7	0.0
Nat. Con.	54.6	121.3	3.9	1.7	52.4	84.3	47.4	44.9
Nat. Free	54.9	118.2	4.2	0.0	49.0	85.2	47.5	0.0

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[a] State Con. refers to Iowa state model constrained to historical.

[b] State Free is the Iowa state model with no acreage constraints.

[c] Nat. Con. is Iowa outcome from historically fixed national model.

[d] Nat. Free is Iowa outcome from acres unrestricted national model.

Table 10. Dual solution (rent) analysis of state model

			Constrained Base				Unconstrained			
Item		unit	#'s of units (1000s)	$/unit	$$10^6$ total rent	share	#'s of units	$/unit	$$10^6$ total rent	share
Land:	1	acre	2898	139	430		2998	115	333	
	2	acre	5887	136	801		5887	114	671	
	3	acre	6826	135	922		6826	109	744	
	4	acre	1980	127	251		1980	99	196	
	5	acre	0	167	0		0	122	0	
	6	acre	785	100	79		785	7	5	
	7	acre	7650	128	979		7650	101	773	
	8	acre	786	122	96		786	49	39	
	9	acre	69	122	9		69	49	3	
	Total		26881		3539	0.95	26881		2764	0.69
Machine:										
	2-F	hour	101	103	10		101	0	0	
	10-S	hour	1963	7	14		1963	53	104	
	13-S	hour	2	80	0		1847	46	85	
	Total				24	0.01			189	0.05
Labor:	Fall	hour	29000	4	116		29000	4	116	
	Spring	hour	25000	4	100		25000	4	100	
	Summer	hour	21000	4	84		17000	0	0	
	Total				300	0.08			216	0.06
Base:	Barley	acre	0	26	0		0	67	0	
	Corn	acre	13633	31	423		13633	67	913	
	Oats	acre	28	4	0		28	67	2	
	Wheat	acre	55	22	1		55	67	4	
	Sorghum	acre	12	21	0		12	67	1	
	Total				425	0.11			920	0.23
Plantable Base(after CRP by soil):										
Corn 1		acre	1407	34	48		1407	29	41	
Corn 2		acre	2932	9	26		0	0	0	
Corn 3		acre	2840	1	3		0	0	0	
Corn 7		acre	10	89	1		0	0	0	
	Total				78	0.02			41	0.01

Table 10. Continued.

		Constrained Base				Unconstrained			
Item	unit	#'s of units (1000s)	$/unit	$$10^6$ total rent	share	#'s of units	$/unit	$$10^6$ total rent	share
Allowable base reductions:									
Barley	acre	0	4	0		0	0	0	
Corn	acre	1849	0	0		0	0	0	
Wheat	acre	41	9	0		0	0	0	
Oats	acre	21	27	1		0	0	0	
Total				1	0.00			0	0.00
Allowable land retirement:									
4	acre	990	0	0		990	2	2	
6	acre	392	28	11		392	93	36	
8	acre	393	5	2		393	55	22	
9	acre	35	5	0		35	51	2	
Total				13	0.00			62	0.02
CRP Cur.($69)	acre	80	-74	-6		80	-48	-4	
Fut.($100)	acre	3218	-62	-199		3218	-57	-183	
Total				-205	-0.05			-187	-0.05
Crop min. Hay	acre	1200	-171	-205		0	0	0	
Oats	acre	1000	-118	-118		0	0	0	
Sorg	acre	670	-206	-138		0	0	0	
Total				-461	-0.12			0	0.00
Net Grand Total				3713	1.00			4005	1.00

Intermodel dual solution inconsistency and lack of correspondence

Relevant dual variables for all but the refined model of Figure 5 were compiled and compared. The Iowa state model represents the entire state as one large farm and dual variables can be read from the solutions. In the national model, production in Iowa is covered by portions of several producing areas and overlying market regions. For the national model dual variables for the state of Iowa were not readily obtainable since the complex weighting schemes involved in estimating producer surplus cannot be manually obtained. Therefore, comparisons in this section are more between constrained and free versions of the same model than between models.

The dual solution summary for the state model is given in Table 10 for both the constrained and free solutions; both rent totals and marginal resource values are reported. The constrained version involves maximum allowable base reductions for CRP or other conversions and minimum acreage requirements for some crops. These artificial constraints reduce the rent attributable to the base acreages by one-halve while increasing the implied land rent by nearly 50 percent as compared to the free solution. In the free case the marginal value of base acres for all crops is the same, indicating that at the margin the ideal use of any base acreage is conversion to CRP at an annual rent of $67. Availability of land group five would enable substitution for more productive land groups in meeting the CRP requirement and that gives this land a high marginal value despite its lower quality (productivity for cropping).

It must be noted that the dual solution is extremely sensitive to constraint values and that the rent generated must not be confused with any implications about supportable levels of fixed costs on the part of the producers. Only if a constraint is actually binding will a positive marginal value and rent exist. For example, an increase of one unit of a resource (say from 1,000,000 to 1,000,001) might lead to it being non-binding and rent dropping from $100 million to zero. At the same time another constraint would experience a similar drastic increase in rent. The marginal values are useful for indicating what should be paid for one additional unit of the resource.

The dual solution summary for Iowa from the national model is given in Table 11 and requires careful explanation. The land resource rows in the national model are equalities, requiring that all land must either be cropped, idled with a green cover or enrolled in the CRP. Since idling the land incurs cover crop establishment and maintenance cost additional land actually harms the producers. This results in the negative land rents shown in Table 11 (the best land still gets a positive rent).

Table 11. Dual solution analysis of national model (Iowa area)

	Base constrained		Unconstrained	
	Total	$/unit[a]	Total	$/unit[a]
Land availability at regional level[b]:				
dryland for cropping	-129809	-6.1	-144651	-5.5
land for irrigation	-576	-4.8	-485	-2.1
land for grazing	0	0.0	0	0.0
Sub total	-130386	-6.1	-145136	-5.5
Land use constraints at regional level[c]:				
minimum crop acreages	-625274	-29.9	0	0.0
minimum irrigation	-713	-8.1	-1022	-11.4
minimum terraces	-21171	-10.5	-19788	-9.8
minimum grazing	0	0.0	0	0.0
minimum CRP enrollment	0	0.0	0	0.0
Sub total	-647159	-27.6	-20811	-9.9
Maximum tillage practices (acres):				
conservation	187370	16.4	0	0.0
zero	18810	19.3	0	0.0
Sub total	206180	16.6	0	0.0
National land use restrictions[c]:				
minimum irrigation	0	0.0	0	0.0
maximum private pasture	0	0.0	0	0.0
Sub total				

---

[a] Marginal values calculated by dividing ARIMS summary table for surplus by acreage values in summary tables 3 and 9.

[b] All land resource rows are equalities, requiring that all land either be cropped, in CRP or in green cover.

[c] Minimum crop and irrigated acres assumed equal to endogenous portions.

TABLE 11. Continued.

	Base constrained		Unconstrained	
	Total	\$/unit[a]	Total	\$/unit[a]
Allowable land conversion:				
dry to irrigated	17	0.5	19	0.5
irrigated to dry	0	0.0	0	0.0
wet soil now cropped	0	0.0	0	0.0
land converted to CRP	0	0.0	0	0.0
Sub total	17	0.5	19	0.5
Supply of surface irrigation water	0	0.0	0	0.0
Land allowed to exceed erosion level	0	0.0	0	0.0
Total	-571349	-21.5	-165927	-6.2
Individual land quality value (average of dry and irrigated)[b]:				
1		1.7		1.3
2		-4.4		-5.1
3		-7.8		-8.2
4		-9.5		-11.1
5		0.0		0.0
6		-11.1		0.0
7		-6.9		-7.7
8		-9.3		0.0
Average		-4.0		-4.5

It is noteworthy that implied land rent in Iowa via the national model is actually decreased by taking out the crop acreage constraints, though the negative producer surplus in the free solution is only 30 percent that of the constrained solution. Apparently, the crop acreage constraints increase the value of Iowa farmland, a conclusion consistent with interpreting the constraints as simulating commodity program subsidy effects on land resource allocation.

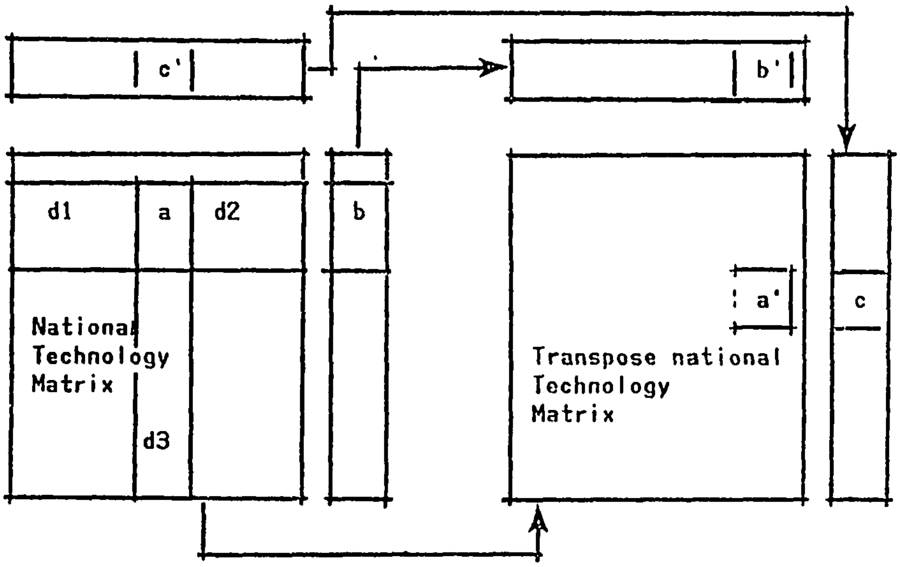
Financial production incentives and possible intermodel consistency

The production incentives existing in each model and those required to make the constrained solutions optimal were tabulated to the extent possible. From these incentives an attempt to develop intermodel correspondences was made. This development was difficult since the methods followed the simple results of Chapter IV while the models involve more complex structure. In particular, the constraint subsets in these applications are only a portion of the overall model constraint sets. Since the dual solution involves the inverse of the entire primal technology matrix the exact correspondences between individual activities and constraints depend on the entire model structure. This problem is illustrated in Figure 8.

National Cost Minimization Model

Primal: Min $C'X$
s.t. $AX \geqslant B$

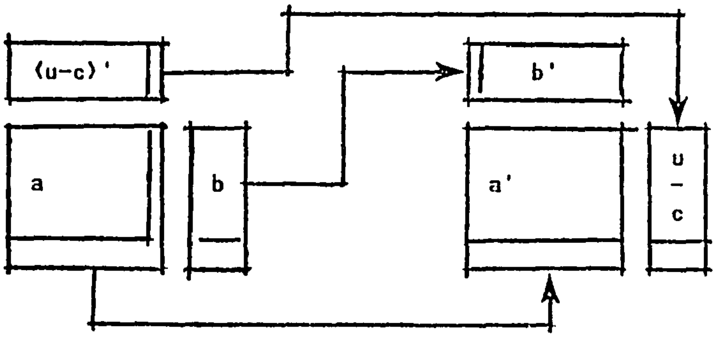
Dual: Max $B'U$
s.t. $A'U \leqslant C$



Regional Profit Maximization Model

Primal: Max $(u-c)'x$
s.t. $ax \leqslant b$

Dual: Min $b'v$
s.t. $a'v \geqslant (u-c)$



Fundamental Lemma of Linear Programming: $C'X < U'AX < U'B$

Equilibrium Theorem of Linear Programming: $C'X^* = U^{*\prime}AX^* = U^{*\prime}B$

Implied solution: $U^* = A^{-1}C$ and $X^* = (A^{-1})'B$

Figure 8. Schematic intermodel primal-dual correspondences

A national cost minimization model is given in the top portion of Figure 8. A regional part of this model consists of the components a, b and c. In the simplistic ideal model example of Chapter IV the national primal model components d1, d2 and d3 would be equal to zero; in the model used in this study they contain coefficients. The problem is further complicated in that the regional profit maximization model is augmented by additional constraints and activities not occurring in the national model as shown by the interior borders in Figure 8. The lower case a, b, c and u letters represent the regional subsets of the national A, B, C, and U. However, in the models of this study even these subsets are not consistent due to differing technology inclusions, overlapping regions, etc. This implies that even if (u-c) of the regional primal model came from the national models as shown in Chapter IV, exact consistent correspondences cannot be found unless aggregators are homothetic.

Implied state model production incentives In this study it is the marginal incentives at the solution point which must be considered as the decision parameters since they are where production decisions are made. To evaluate these incentives requires both model input data (selling prices, production costs, yields, etc.) and solution characteristics such as participation rates, shadow prices and marginal costs. Given the complex model structure isolation of marginal costs from the given solutions was impossible so average costs based on average yields from the solutions are reported. However, average cost is only the same as marginal cost if marginal cost is constant or if average cost is at a minimum point and so the rent (returns) conclusions given below are not strictly valid.

For the state model production incentives are reported only for the original constrained version. Incentives are the same in the free version except that constraints to decisions are removed. In the state model various activities represent different portions of the overall production and revenue generating process. Production activities simulate various methods of producing crops on a per acre basis at a given variable cost. Additional production activities can be chosen to apply chemicals and perform extra tillages if optimal. Program participation activities generate additional revenue for cropping activities while requiring set aside to non-use of some land resources. Table 12 shows various aspects of the incentives apparently existing in the model. Input data to the model construction process included selling prices, loan rates, deficiency payments, diversion payments and set aside requirements as shown in the first eight lines of data in Table 12. Each item in Table 12 is explained in detail and then implications for modeling correspondences are discussed.

Table 12. State model financial production incentives

	BAR	CRN	LHY	NLH	OTS	SRG[a]	SOY[b]	WHT
Selling Price	1.30	1.35	45.30	33.30	1.20	1.20	5.02	2.30
Loan Rate	1.56	1.92	0.00	0.00	0.99	1.82	5.02	2.40
Deficiency payment	1.04	1.11	0.00	0.00	0.61	1.06	0.00	1.98
Diversion payment								
voluntary	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00
mandatory	0.57	0.73	0.00	0.00	0.36	0.65	0.00	1.10
Diversion set aside percent								
voluntary	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00
mandatory	3.00	3.00	0.00	0.00	3.00	3.00	0.00	5.00
Set aside percent	17.50	17.50	0. 00	0.00	17.50	17.50	0.00	22.50
Implied prices								
Total	2.62	3.05	45.25	33.25	1.90	2.90	5.02	4.54
Net(w/set aside)	2.09	2.44	45.25	33.25	1.52	2.32	5.02	3.06
Average cost[c]	1.17	1.14	30.81	44.25	0.78	1.36	2.30	2.55
Marginal revenue (per unit)								
Program	0.92	1.30	14.49	-11.11	0.74	0.96	2.72	0.53
Non-program	0.13	0.21	14.49	-11.11	0.42	-0.16	2.72	-0.25
Marginal returns (per acre)								
Program	59.82	176.80	53.43	-20.00	39.07	86.78	88.67	20.78
Non-program	8.46	23.37	53.61	-20.00	22.18	-14.46	88.67	-11.23
Program returns	51.36	148.43	0.00	0.00	16.89	101.24	0.00	32.00
Returns Rank								
Program	4	1	5	9	6	3	2	7
Non-program	5	3	2	8	4	7	1	6
Overall Rank								
Program	5	1	6	15	8	4	2	11
Non-program	12	9	7	16	10	14	3	13
Participation rate								
Constrained	0.00	100.00	N/A	N/A	0.60	1.50	100.00	0.90
Free	0.00	97.20	N/A	N/A	0.00	0.00	100.00	0.00
Implied Subsidy	2.04	1.09	25.40	91.10	1.99	2.27	0.47	2.53

[a] Sorghum can only be grown in rotation with soybeans or wheat in equal proportions with the other crop in both cases.

[b] Soybean loans are factored into price assuming state participation ratio applies equally to all acres.

[c] Average cost calculated as average over all optimal activity levels.

Production activities generate commodities which via transfer rows are sold at the selling price and crop acres which can be used as a resource by the participation activities. The method of calculating the objective function coefficient for the participation activities depended on whether the loan rate was above or below the selling price. In both cases the revenue is equal to deficiency payment per acre plus diversion payment per acre. Deficiency payment is either actual yield multiplied by deficiency plus loan minus price if loan greater than price or deficiency multiplied by actual yield otherwise; in both cases a participation rate of 100 percent is assumed. Diversion payments are equal to base yield multiplied by diversion payment rate. For soybeans the selling price was adjusted to assume a uniform participation rate for every acre cropped (i.e., the historical participation rate was used) and inclusion of deficiency payment directly, i.e., no separate participation activities were generated. These payment considerations resulted in the "Implied total price" shown in Table 12. A reduction of this price by the total required set aside and diversion resulted in the "Implied net(w/idle)" price.

Average cost per acre for each crop was calculated as the acreage weighted average over all optimal production activities. First, the crop production activities were considered. Since each activity included several crops grown in rotation a linear set of equations was formulated to factor out individual average per acre crop production costs. These were then converted to per unit costs by dividing by average yields. Finally, for each of corn and soybeans all the pesticide application and extra cultivation costs were added up and divided by total production. This resulted in a gross average cost. In reality, it is difficult to correlate the average costs of specific techniques with the marginal costs and returns of participation decisions, etc. This method of average cost calculation yields no information about marginal costs.

Marginal revenues and returns were calculated using ex-poste model solution average production costs rather than marginal costs. The same average cost was assumed to apply equally for both program participants and non-participation. Marginal returns were calculated by multiplying marginal revenue by average yield. The use of average cost and yield instead of marginals biases these marginal return results somewhat; however, neither the direction nor the magnitude of the bias is known. The returns to program participation are the per acre returns differences from participating and non-participating.

The marginal returns per acre are ranked, first for participation or not, and then for the overall decision wherein the producer considers both participation and the crop mix decisions. The actual participation rates occurring in the model solutions are also shown in Table 12. The correlation between returns ranking and program participation is fairly good in the constrained case even though the various constraints in the model impose additional implied incentives.

Implied subsidy rates were calculated by considering whether or not participation occurred, whether or not crop acreage constraints were binding and shadow prices on those constraints (both crop mix and minimum acreages). For barley the small grains mix constraint implied a $2.44 per bushel subsidy in the absence of participation. For corn no acreage constraints were binding and the implied subsidy was equal to implied net price minus the selling price. For hay the crop constraints were binding, but signs differed across crops and in some cases the subsidies implied by the crop mix were offset by the those of the crop minimums and vice versa. For oats, sorghum and wheat acreage constraints were binding but participation did not occur. The soybean subsidy is calculated the same as for corn. There is not a clear explanation as to why with such large subsidies needed for the constrained levels to be optimal that program participation, which would of offset at least part of the required subsidy, did not occur. This result may be due to the relative scarcity of base acres of these crops given the CRP idling requirements.

National model production incentives   The national model has a national level cost minimization criterion. The basis assumption required for interpretation of the model solution is that commodity price is equal to marginal cost of producing the specified level of commodities. This allows interpretation of the shadow prices of the demand constraints as prices received by producers and on which decisions are based. However, choices are constrained by individual minimum crop acreages. To the implied commodity demand constraints can be added the shadow prices of individual crop constraints divided by yield. These implied total marginal prices are then what would be required to cause the constrained solution to be optimal given all else remaining the same.

As explained earlier in the model structure section the national model producing and marketing areas are not by state boundaries. Summary programs prepare state level results reporting tables from the model using fixed (historical) weights for the Iowa portions of the several overlaying producing and marketing regions. However, these tables do not give the same level of detail for the primal and dual solutions at the state level as is available for the national model summary. Therefore, either only approximate averages from the summary tables or representative values from predominate model areas are used in the following analysis.

Summary tables showing the total production cost for each commodity and the marginal demand values were available for both the constrained and free solutions. Average production cost was found by dividing the reported total cost by total production. The implied subsidies due to the binding acreage constraints were calculated by using the actual model solutions and assuming weights for the areas involved. Weights of 0.64, 0.32 and 0.04 were assumed for market areas 12, 15, and 16, respectively. In each of these regions it was determined if the crop minimum acreage constraint was both non-zero and binding so as to determine if the marginal was due to forcing in all those regions or if one or more regions were producing beyond the constraint, in which case the marginal would be zero for the state.

The marginal demand prices and implied subsidy rates were added together to give a "total marginal" incentive as shown in Table 13. Implied returns were calculated as the implied subsidy multiplied by yields; however, since average cost is greater than marginal cost this cannot represent an equilibrium solution and so the "rent" calculation is invalid.

Note that implied rent ranks are different than in the state model. Here they are very similar in magnitude across crops, indicating that perhaps some common factor is causing a generic acreage constraint.

Table 13. Iowa financial production incentives (national model)

	BAR	CRN	LHY	NLH	OTS	SRG	SOY	WHT
<u>Constrained Model</u>								
Average cost[a]	1.56	1.35	29.44	33.09	1.65	1.21	2.61	2.07
Marginal demand value[b] (marginal cost)	0.85	1.13	3.79	0.03	1.45	0.97	2.53	1.63
Minimum acres implied subsidy[c] (marginal cost)	0.46	0.36	16.21	16.21	0.00	0.32	0.25	0.34
Total marginal[d]	1.31	1.49	20.00	16.24	1.45	1.25	2.78	1.97
Returns/acre[e]	-13.65	16.98	-36.82	-28.65	-10.56	3.62	8.11	-4.49
rank	6	1	8	7	5	3	2	4
Implied rent[f]	25.25	43.67	63.22	27.56	0.00	26.98	11.93	15.23
<u>Free Model</u>								
Average cost	1.39	1.36	18.67	0.00	1.57	0.98	2.36	0.00
Marginal demand value (marginal cost)	1.09	1.23	16.25	0.00	1.57	0.91	2.53	0.00
Returns/acre	-16.47	-15.37	-10.16	0.00	0.00	-5.56	-8.08	0.00
rank	6	5	4	?[g]	1	2	3	?[g]

---

[a] Average cost is total cost divided by total production.

[b] Marginal demand value is shadow price on demand constraint.

[c] Marginal cost of minimum acres is shadow price divided by average yield.

[d] Total marginal is sum of shadow price of demand and acres constraint.

[e] Returns are total marginal minus average production cost.

[f] Implied rent is implied subsidy multiplied by yield.

[g] Since production equal zero, as are returns, the rank is not known.

For nearly all commodities in both model versions the implied rent is negative. This is due to the demand prices being determined by national equilibrium supply and demand conditions while average cost is determined by local technology and the presence of non-optimal forcing constraints. As an example, even if crop production costs were lower in the rest of the nation, or for another technique in Iowa, giving a low marginal commodity value, production must still occur to meet the various constraints.

Implications for correspondences The closest correspondence available is in the implied prevailing subsidy rates in the models. If marginal cost information were available and if it were similar across the models (likely since nearly the same technology, resource and constrained acreages are in both) then it seems that rent rankings are nearly the same.

Several implications for model structure and solutions can be drawn from this exercise even if clear correspondences cannot be shown. If constraints in the national model force a crop in to meet non-demand constraints (in which case the marginal cost of production is lower elsewhere) and constraints accomplishing more or less the same also exist in the state model then links are clear--primal outcomes both equal the effective constraint levels while dual impacts are in the same direction but of differing magnitude depending on the deviation of implied price from marginal cost existing in the models. However, if this is the case, then impact estimates due to policy implementation are biased since no quantity adjustment is allowed: biased downward on the primal activities side and upwards on the dual resource evaluations.

Due to linearity the profit model would naturally choose only to produce the most profitable crops. Constraints would be required to force in the less profitable crops. The shadow price on these constraints would be equal to per acre profit differences which could be converted to implied needed price subsidies by dividing by yield. For the state model this would then yield an indeterminate solution as shown in Chapter IV. If non-linear marginal cost could be assumed, due to linearized steps in the LP, etc., a combination of subsidies and/or penalties could be imposed to get the solutions consistent. However, non-linearity of the forms usable in an LP implies aggregation inconsistency and clear analytical forms seem to be non-derivable given the complex and differing nature of these models.

The Refined State Model and Correspondence Implications

Model changes

Three sets of constraints in the state model were taken out for this exercise: the plantable base resources, the proportional crop mix for hay and small grains and the allowable base reductions. The reasoning behind each of these changes is discussed separately below.

The purpose of the plantable base acreages constraints in the original model as obtained for this study was to force all the base acres not used for CRP (previous and current signup assumptions) to be used in program participation or face a penalty. The penalty was equal to the discounted Net Present Value (NPV) of expected loss of program earnings in future years. As equalities in the model these constraints were distorting the dual solution tremendously so in the constrained version of this study they were changed to upper bounds on plantable base acres. This change had little impact on the primal solution but greatly improved the dual. The intent of this constraint could be maintained by having one constraint for each crop requiring all the base acres to be used, enrolled in CRP or idled with a penalty. However, the CRP is already fixed in total and proportionally for individual crop base reductions. Program returns are positive for all crops and so the base will surely be used without these constraints. In any case, only the corn base has a non-zero penalty for non-use. Actual idling of base acreage in other crops in both the constrained and free runs was small, and probably necessary to meet machinery limitations, etc., due to the complex model structure.

The "ALLOC" proportional crop mix acreage constraints impose the assumptions of homotheticity on the hay and small grains aggregates; this is an assumption that seems particularly undesirable given the intertwined commodity and resource policies studied here. The minimum crop acreage constraints will accomplish the goal of non-specialization. Relatively few acres are involved in these constraints and so neither the primal nor dual impacts of this change should be expected to be large.

Finally, the allowable base reductions constraints were taken out, allowing the model complete freedom for idling all of any crop vase via CRP enrollment or set aside with a penalty. A clear justification for the original constraints was not available.

Results

The refined constrained model produced results (both primal and dual) amazingly similar to the original version (compare Tables 14 and 15 to Tables 8, 9 and 11). On the primal side this similarity was to be expected given the linearity of the model and the tendency to specialize in the most profitable crops subject to the net constrained diversion to other crops which remained essentially unchanged. On the dual side clearer links between production incentives and constrained state and national outcomes were expected. However, as explained earlier, other differences in model structure as well as empirical content stand

Table 14. Refined model outcome comparison (primal solution)[a]

	BAR	CRN	LHY	NLH	OTS	SRG	SOY	WHT
Acres(1000s)	4.0	9837.0	1200.0	77.0	1000.0	670.0	8728.0	100.0
% diff.	0.0	0.2	0.0	-88.0	0.0	0.0	6.8	-7.4
Output(10^6)	0.2	1356.6	4.1	0.1	52.8	60.2	281.5	3.7
% diff.	-7.7	2.0	-6.8	-91.3	0.6	-0.7	5.8	-11.6
Yields	60.0	137.9	3.4	1.3	52.8	89.9	32.3	37.4
% diff.	-7.8	2.1	-7.6	-27.8	0.0	-0.6	-1.0	-4.6
Average cost	0.9	1.1	35.0	65.0	0.5	1.5	2.1	2.5
% diff.	-19.6	-6.1	13.6	47.7	-34.6	13.2	-9.1	-1.6
Marginal revenue (per unit)								
Program	1.15	1.37	10.30	-31.70	1.01	0.78	2.93	0.55
Non-program	0.36	0.28	10.30	-31.70	0.69	-0.34	2.93	-0.21
Marginal returns (per acre)								
Program	69.00	188.92	35.23	-41.20	53.33	70.12	94.49	20.57
Non-program	21.60	38.61	35.23	-41.20	36.43	-30.56	94.49	-7.85
Program returns	47.40	150.31	0.00	0.00	16.90	100.68	0.00	28.42
Returns rank								
Program	4	1	6	8	5	3	2	7
Non-program	5	2	4	8	3	7	1	6
Overall rank								
Program	5	1	10	15	6	4	2	12
Non-program	10	7	1	16	9	14	3	13
Participation rate	0.00	1.00	0.00	0.00	0.02	0.00	1.00	0.00
Implied subsidy	2.39	1.09	78.34	46.64	1.79	2.05	0.47	3.07
% diff.	17.16	0.00	208.43	-48.80	-10.05	-9.69	0.00	21.35

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[a] Percentage differences are with the original constrained model as base.

in the way of clear derivable correspondences. Crops being producible only in rotations as a multi-product process also limit the correspondences derivable for individual crop acres and commodities. Though the shares of individual resources in total imputed returns change for the refined model a judgement as to which of the original or refined versions give the best welfare estimates is not made.

Table 14 compares the primal solution, prices and returns of the refined constrained model to the original. Acreages of barley, legume hay, oats and sorghum are unchanged since they remained at the original binding minimum levels. Corn yields and production increase about two percent with acreage nearly constant, indicating a shift to more productive land and/or techniques. For soybeans the acreage increase is greater than the production increase, indicating a shift to poorer land. All other crops show a decline in yields. Acreage of non-legume hay suffered the greatest decrease (88 percent) along with a 28 percent yield decline. Barley appears to have suffered the most significant resource use shift since its yield fell the most.

Despite the shift to poorer lands and the concurrent yield declines the average production cost declined for barley, wheat and oats. The most drastic average cost changes were an increase of 48 percent for non-legume hay and a decrease of 35 percent for oats. With constant prices and commodity program parameters these average cost changes translate directly to changes in returns per acre.

The impact of the alternative method of constraining the solution can be seen in the "Implied Subsidy" changes from Table 5 to Table 14 (though marginal costs are unknown their changes are sort of implied here). The lower yields for barley, legume hay and wheat are directly compensated for by increases in the implied subsidy rates of 17.6, 208.4 and 21.4 percent, respectively. An increase in the implied subsidy for non-legume hay would also likely have occurred if not for the drastic acreage decrease.

The overall difference between the pattern of implied subsidies in the state models and the national was estimated in the following manner. First, for both the original and the refined state models the percentage differences in subsidies between them and the national were calculated for each crop. For each state model version the sum of squared percentage differences was calculated, the square root taken and the result divided by the number of crops. This calculation gave an "average" percentage difference of 137 for the original model and 147 for the refined--apparently the original more closely follows the national pattern.

A comparison of the dual solutions of the refined and original state models is given in Table 15. First, note that shares of returns imputed to resources are affected significantly. The land rent share decreases with the change going to an increase in the program base acreage resource share. This likely occurred because crops for which base acreages were not available or as profitable (hay and small grains) were being required in fixed proportions, etc., creating an artificially scarce land base in the original model.

The CRP enrollment constraint is more restrictive since its contract payment was unchanged while crop base acreages became more valuable. The share accruing to the individual crop minimum acres is smaller in the refined model. Note that the "ALLOC" crop mix constraints in the original model had no non-zero RHS in either model version and so receive no rent share, despite their large shadow prices (up to $150 per acre).

Individual land quality marginal values changed drastically as compared to the original solution. The best land received a five percent increase in value while marginal values declined for all other land groups with the poorest lands receiving a 13 percent decrease. Marginal values for land and machinery were unchanged. Except for sorghum, the marginal values of all base acreages are equal, implying a common use for the marginal acre (CRP enrollment).

Table 15. Iowa model dual (rent) comparison of refined to original

			Constrained Base				Refined Constrained			
Item		unit	#'s of units (1000s)	$/unit	$$10^6$ total rent	share	#'s of units	$/unit	$$10^6$ total rent	share
Land:	1	acre	2898	139	430		2998	146	422	
	2	acre	5887	136	801		5887	130	764	
	3	acre	6826	135	922		6826	125	851	
	4	acre	1980	127	251		1980	116	230	
	5	acre	0	167	0		0	158	0	
	6	acre	785	100	79		785	65	51	
	7	acre	7650	128	979		7650	117	894	
	8	acre	786	122	96		786	106	83	
	9	acre	69	122	9		69	106	7	
	Total		26881		3539	0.95	26881		3302	0.87
Machine:										
	2-F	hour	101	103	10		101	103	10	
	10-S	hour	1963	7	14		1963	7	14	
	13-S	hour	2	80	0		1847	78	0	
	Total				24	0.01			189	0.05
Labor:	Fall	hour	29000	4	116		29000	4	116	
	Spring	hour	25000	4	100		25000	4	100	
	Summer	hour	21000	4	84		21000	4	84	
	Total				300	0.08			300	0.08
Base:	Barley	acre	0	26	0		0	52	0	
	Corn	acre	13633	31	423		13633	52	702	
	Oats	acre	28	4	0		28	52	1	
	Wheat	acre	55	22	1		55	52	3	
	Sorgh.	acre	12	21	0		12	52	1	
	Total				425	0.11			707	0.19
Plantable Base(after CRP by soil):										
Corn 1		acre	1407	34	48		0	0	0	
Corn 2		acre	2932	9	26		0	0	0	
Corn 3		acre	2840	1	3		0	0	0	
Corn 7		acre	10	89	1		0	0	0	
	Total				78	0.02			0	0.00

Table 15--Continued.

		Constrained Base				Refined Constrained			
		#'s of units		$10^6 total		#'s of units		$10^6 total	
Item	unit	(1000s)	$/unit	rent	share		$/unit	rent	share
Allowable base reductions:									
Barley	acre	0	4	0		0	0	0	
Corn	acre	1849	0	0		0	0	0	
Wheat	acre	41	9	0		0	0	0	
Oats	acre	21	27	1		0	0	0	
Total				1	0.00			0	0.00
Allowable land retirement:									
4	acre	990	0	0		990	1	1	
6	acre	392	28	11		392	51	21	
8	acre	393	5	2		393	11	4	
9	acre	35	5	0		35	11	0	
Total				13	0.00			26	0.01
CRP Cur.($69)	acre	80	-74	-6		80	-64	-5	
Fut.($100)	acre	3218	-62	-199		3218	-64	-206	
Total				-205	-0.05			-211	-0.06
Crop min. Hay	acre	1200	-171	-205		0	0	0	
Oat	acre	1000	-118	-118		0	0	0	
Sor	acre	670	-206	-138		0	0	0	
Total				-461	-0.12			0	0.00
Net Grand Total				3713	1.00			3808	1.00

Marginal values on crop mix constraints in the original model were $-132.8, 77.2, -144.1, 13.0, and -114.9 for barley, legume hay, non-legume hay, oats and wheat respectively. Changes in shadow prices on CRP land group allocations were also rather drastic. The percentage changes for land groups one to six were: 216.3, 11.7, 6.0, -100.0, 40.9(zero in original to 40.9 in refined), and -100.0.

Despite the implied subsidy patterns of the refined model looking worse than in the original, the changes seen in the overall dual solution are expected. The changes in the dual solution indicate more diverse production opportunities with a wider spread of costs (indicated by the wider spread in resource values across productivity levels). The result is a model more responsive to policy intervention and hence larger primal (smaller dual) impact estimations.

CHAPTER VI.

CONCLUSIONS

In this work the problem of attaining both primal and dual consistency within and between models of a system has been addressed. Strict aggregation consistency theorems have been evaluated with regard totheir implications for regional/national natural resource policy modeling. It was noted that few consistent non-linear aggregators for which data are available in the appropriate aggregate form are available that are usable in national and regional natural resource programming models. In the case of linear functional forms for which the available aggregate data are consistent the resulting linear model may be behaviorally inconsistent.

Traditionally, linearity in natural resource models has resulted in behavioral inconsistency which leads to the use of artificial constraining methods to guide the primal solution. Artificial constraints often result in dual solutions uninterpretable from a resource market view. Methods of partially overcoming these modeling aggregation and behavioral consistency problems have been suggested in this study. These findings are categorized in more detail in the remainder of this chapter.

The conclusions arising from this study can be classified as follows: the strict consistency propositions arising from theory; difficulties in satisfying the theoretical propositions in applied work; and interpretive guidelines for the use of artificial constraining procedures in applied work. Each of these headings will be addressed separately below. Finally, some suggestions for further study of the problem are given.

Strict Aggregation and Behavioral Consistency Propositions

A. In the absence of any restrictions on the values that variables may take such that consistency depends solely on functional forms.

   1. If both dependent and independent variables are assumed to be simple sums, the individual functions being aggregated must be linear with identical slopes (Green, 1964).

   2. If either the dependent or independent variable is allowed to be a weighted sum the functions must still be linear and the ratio of slope coefficient to weight must be constant across all functions (Green, 1964).

      a) In consumption functions the ratio of income to marginal propensity to consume (the weight) must be constant across all households for the aggregate function to be consistent (Green, 1964).

  - b) Production functions must be linear and homogeneous with intercept terms equal to zero and exhibit parallel hyperplane isoquants, identical across all firms (Green, 1964).

3. If log forms of non-linear functions can be taken then the log forms can be treated as linear functions for the purposes of aggregation (Green, 1964).
4. For arbitrary second degree polynomials complex weighting restrictions are essentially that all own variable second degree term coefficients and all cross term coefficients be equal across all functions (Green, 1964; Theil, 1954).

B. Assuming that the variable value restrictions arising from the optimization FOC of a single decision maker separability and/or homotheticity may allow aggregation of the overall decision to a two (or more) stage budgeting procedure.

1. For decisions about the components of each group aggregate to be independent of components of other aggregates, i.e., for the second stage of the budgeting procedure to be consistent, weak separability of the groups is required (Green, 1964).

2. For the overall two stage budgeting (aggregation) procedure to be consistent (i.e., give same result as original problem and have the product of each group price and quantity index equal group expenditure).
    a. If only two aggregates are involved in the decision function, weak separability between the two groups of micro variables is sufficient (Green, 1964).
    b. If more than two aggregates are involved either the overall decision function is strongly separable in all the micro variables or each group quantity index is homothetic (Blackorby et al., 1978).

C. For aggregation across firms under a linear rule where total output is equal to the sum of individual outputs and total cost is equal to the sum of firm costs.

1. The aggregate cost function must be a linear transformation of a homothetic technology and if it is to satisfy actual properties of micro level cost functions then the following (Chambers, 1988).
    a) Production functions must be linear homogeneous.
    b) Each firm level marginal cost must equal aggregate marginal cost.

        c) Each firm level marginal cost must be independent of firm level output.
        d) Marginal costs must be identical across firms.
        e) Aggregate marginal cost must be independent of aggregate output.

D. If a non-linear aggregation rule across firms is allowed for output then marginal costs across firms need not be identical (Chambers, 1988); however, the following two restrictions apply.

    1. Each firm level marginal cost must be constant and equal to aggregate marginal cost multiplied by the appropriate weighting factor (Chambers, 1988).
    2. Aggregate marginal cost must still be constant and independent of output (Chambers, 1988).

E. If non-linear aggregation rules for both output and costs are allowed then the following conditions are needed.

    1. Aggregate marginal cost need not be independent of aggregate output but aggregate cost must be additively separable in firm level outputs (Chambers, 1988).
        a) Marginal cost is not constant either for the aggregate or individual firms (Chambers, 1988).
        b) Homothetic production structures can be. represented without imposing constant returns to scale (Chambers, 1988).

c) Firms can have differing technology, even if the fixed or constant cost term of the aggregate is forced to zero for consistency with micro level cost function properties; however, each firm level production function is a transform of the same linearly homogeneous production function and input requirement sets are still required to be parallel across firms (Chambers, 1988).

F. For linear programming model aggregation consistency the following matrix conditions are required (Hazell and Norton, 1986).

1. Technological homogeneity (identical technology matrices).
2. Pecunious proportionality of net returns expectations (objective functions differ only be a multiplicative constant).
3. Institutional proportionality (RHS vectors differ only by a multiplicative constant).
4. Strict sufficiency requires that the average or representative firm is the arithmetic mean (the sum of constants for the RHS vector differences must be equal to one).

G. For use of price and quantity indices in welfare analysis it is required for a "true" cost index that the production or utility function being optimized by the decision maker be homothetic (Diewert, 1981).

1. If the decision function is not homothetic then for the various price and quantity indices suggested by other authors only upper or lower bounds or differences to the "true" index can be found (Diewert, 1981).
2. If functional forms capable of providing second order approximations to arbitrary aggregators can be found which are homothetic then the price and quantity indices will be exact (Diewert, 1981).
3. In some cases exact indices can be found for aggregators which are second order differential approximations to arbitrary twice continuously differentiable linear homogeneous aggregator functions; in this case the production or utility functions need not necessarily be homothetic (Diewert, 1981).

H. If national competitive equilibrium and regional or firm level natural resource policy programming models satisfy consistent aggregation with linear aggregators, i.e., constant marginal cost, then behavioral inconsistency may occur (see Chapter IV).

1. Linear aggregation rules give consistent results but at the sacrifice of consistent behavioral outcomes.
    a. Restrictions on allowed technology and decision rules are such that the aggregate model automatically performs the same as the total of the individuals, but this is due to the assumptions or requirements of homogeneity.
    b. Policy impact estimation under consistent linear aggregation rules give unuseful information.
    c. Profit maximization with technology consistent with these rules gives either indeterminate solutions or production up to the limit imposed by available resources.
2. Non-linear aggregation rules are difficult to incorporate in natural resource policy models.
    a. Simple non-linear rules which can be incorporated still yield inconsistent results or else require the assumption of unrealistic behavior on the part of decision makers.
    b. Complex non-linear rules require non-linear programming model specifications where convergence to an optimal solution is difficult or else incur the curse of dimensionality when linearized in an LP.

3. If artificial specialized model constraining methods are used to enforce primal model consistency then dual solutions will be inconsistent unless special interpretations can be made (see examples in Chapter V).
    a. Inconsistent dual solutions result in inconsistent resource market policy impacts.

Conflicts in Satisfying Strict Propositions in Applied Work

A. Commodity and resource policy programs are linked; current levels of commodity subsidies result in positive marginal profits but program parameters are somewhat dependent on aggregate outcomes and interest is building for adjusting the parameters to achieve a competitive equilibrium.

B. Modeling capacity limitations result in economists using less than ideal approximate analytical tools; these tools are inconsistent from a behavioral and aggregation standpoint and given the complexity of current policy interventions provide inconsistent results when compared to each other.
    1. Cost minimization and profit maximization models are able to contain very detailed resource and technological information but are limited in their ability to simulate economic behavior.

a) Cost minimization models simulate a long run competitive supply equilibrium for a given demand vector under the assumption of non-existence of government commodity price distorting policies.

b) Profit maximization models assume perfectly elastic product demand and factor supply at fixed prices.

c) Both cost minimization and profit maximization models may give inaccurate welfare impact estimates since demand (price or quantity) is fixed (Taylor et al., 1977).

d) Both types are also often inadvertently converted to national planning orientations by the imposition of policy goals in the form of constraints (Hazell and Norton, 1986).

e) These models involve aggregation and so introduce inconsistency given a heterogeneous real world and introduce behavioral inconsistency by virtue of only partially simulating the equilibrium determining processes of economy.

2. Market equilibrium models exist but in general involve a much higher degree of aggregation than non-equilibrium models; they do give correct welfare impact

estimations but computational capacity requirements are large.

a) The criterion utilized can be either surplus maximization or market rent minimization but integrability of demand functions is required for valid welfare interpretations.

b) Approaches of quadratic programming (linear supply and demand), LP linearization of the surplus terms, hybrid econometric market and LP supply model systems, and full non-linear programming models are all possible but all encounter the curse of dimensionality in model capacity; specialized functional form restrictions are generally required to guarantee global optimality in the solution process.

c) These models involve even higher levels of aggregation and there is no guarantee that the macro-level behavior will be consistent with the total of the micro unit behavior being aggregated.

3. Resource directive, price directive and discrete alternative programming decomposable methods allow a larger more ideal model to be divided into manageable parts, each of which can be solved separately.

   - a) Separability between components of the large model is required.
   - b) Specialized functional form restrictions are needed to guarantee iterative convergence to an equilibrium.

4. Use of a system of farm models with numerous price parametric solutions to generate data for use in econometric supply and demand estimations can be used but a very large manpower requirement.
   - a) Since obviously, not every farm in the sector can be modeled explicitly, the choice of average or representative farms involves aggregation error.
   - b) The large amount of model solving and calibration work involved in the parametric solution space generation is very specific to the set of price and policy parameters and likely will need repeating for each new policy application.
5. Other approaches such as non-linear econometrically estimated programming and endogenous technology coefficients within a stable basis involve even higher levels of aggregation than the other models cited above and are also limited in their ability to simulate complex policy linkages.

C. Modeling capacity and ability limitations have generally been such that economists have been satisfied to compare several "good" policy parameters rather than optimize and search for the "best" parameters.

   1. No measures can usually be given of how far the chosen good options deviate from the best option; hence, the ranking of modeled good alternatives may even be wrong.
   2. Mathematical difficulties are encountered when endogenous policy parameters are optimized simultaneously with decision maker behavior since a tradeoff programming problem results.

D. Empirical considerations limit what can be done even if theoretical propositions can be satisfied in model specification.

   1. Data availability by differing spatial and temporal scale require weighted averaging schemes for coefficient development.
   2. Real world multi-product externalities such as crops being grown in rotation, fixed factors being used for different products at different times, etc., limit the clear behavioral correspondences which can normally be derived between different models.

3. Capital considerations limiting shortrun flexibility of choice such as risk, servicing of outstanding debt, etc. also impose limits on the interpretations available from model solutions when these factors are captured by means of artificial constraints.
4. Assuming non-homotheticity and using endogenous weights would improve analytical results; however, if one had sufficient data and model specification for endogenous weights, one would already know what the policy estimation outcomes would be.

Interpretive Guidelines for Constraining Procedures

A. Aggregation consistency theorems are available and can serve as useful guidelines for constructing aggregate models and for the type of qualifiers which need to be attached to aggregate results and to the corresponding welfare impact estimates.

1. Even though theorems for consistent aggregation are available, satisfaction of their requirements may not make sense in empirical economic analysis; the required restrictions on technology and preferences are generally not believable in an empirical sense.
2. For policy analysis, satisfaction of the stringent consistency theorems implies an absence of differential impacts across and within groups of agents and/or regions and so analysis is not so useful.

3. Since the satisfaction of the theorems are not so believable in an emperical sense, failure to satisfy strict consistency with chosen modeling structures should not be considered so serious.

4. The theorems do provide useful guidelines on the implications of choosing among alternative less-than-strict procedures when empirical models are being constructed.

B. The use of systems of models where the lower dimension models are components of the higher dimension model can be justified for several alternative scenarios of model structure and consistency. This is important in empirical national/regional and/or aggregate/firm level modeling systems for policy analysis.

1. Suppose that either consistent non-linear aggregation forms are used or that the issue of aggregation consistency is overlooked such that with the chosen models the first and second order conditions for optimization are satisfied. In this case intermodel pricing correspondences can be solved for so that the lower dimension models give the same primal and dual solution results for that entity and level of aggregation as are given by the higher dimension model.

2. If the chosen models involve linearity or are abstractions of reality to such an extent that artificial constraining methods are required to condition the primal solutions of the models in the system, information from the

corresponding dual solutions can be used to adjust both the constraint levels and the pricing rules incorporated in the objective functions such that consistent solutions (and policy impacts) can be estimated.

3. In cases where model structure is sufficiently complex that analytical solutions are difficult to obtain and interpret some guidelines for intermodel correspondences can be obtained from simpler models. Synthetic models of a simpler nature could be constructed and solved with numeric procedures to compare the effects of alternative model structures when analytical results are difficult to obtain.

4. These results imply that regional models can be used for natural resource policy analysis in the United States in a manner such that their solutions will be consistent with national analysis (which may not be performed). This means that models can be tailored to regionally specific issues while interregional externality issues are accounted for and that overall analysis will be more efficient.

C. Dual solution information and intermodel correspondences can be used to improve the resource market impact estimations coming from models developed independently but used for analysis of a common policy problem.

1. When artificial model structure in the form of constraints must be used to achieve primal solution consistency the simplest possible set of constraints for achieving this outcome ought to be used so as to more fully enable interpretation of resource market implications via dual solution analysis.
   a. With simple constraining methods shadow prices on constraints can be converted to readily understandable units such as "implied commodity subsidies".
   b. Each non-zero RHS binding constraint will have a rent share attributed to it; reducing the number of these constraints results in a more correct imputed rent for the real resources specified in the model and in easier interpretation of the rents of the remaining artificial constraints in terms of resource market implications.
   c. Constraints with zero RHS which are binding receive no rent share but greatly distort the rent shares of the real resources as shown by the applied example of the previous chapter.

   d. When artificial constraints are incorporated in the model structure their impacts in both the baseline and policy scenario primal and dual solutions must be carefully examined.
      i. When these constraints were binding in both the baseline and scenario solutions it is incorrect to attribute zero impacts to the policy imposition.
      ii. The more complex the nature of the constraining structure the more difficult it is to examine these considerations.
   e. Characteristics of the dual solution should be examined to see if they are typical of an equilibrium situation; if pressures for further primal solution adjustment are manifest in the dual solution either the model should be respecified and solved or the resource market impacts adjusted for the implied economic incentives shown in the dual.

2. Some authors have suggested more complex multi-crop or whole farm structure for the activities in aggregate sector models (for example, McCarl, 1982); the results of this study indicate that such structure will result in uninterpretable dual solution values which if

ignored in the resource market impact estimations result in biased welfare estimates.

3. To the extent possible any model which represents only a portion of an overall economy or sector should be validated via behavioral links to a model of broader scope and vice versa.
    a. Such a validation may be achievable without impacting the primal solutions prevailing in the absence of validation but will enhance greatly the resource use and value impact interpretations possible with the model.
4. Measures of the differences in effectiveness of alternative intermodel correspondences should be evaluated and refined; currently accepted measures exist for primal solution variables but are lacking for "implied subsidy rates", etc., associated with the dual solution interpretation.

Suggestions for Further Work

The empirical application of this study indicated several areas where further work would be useful. First, difficulty was encountered in comparing and judging which alternative model structure gave the "best" dual solution. The dual solutions were different but judgement of which was better would seem to require comparison to empirical rent and/or returns data for a calibrated baseline scenario. Second, comparison of economic production incentives patterns of two alternative regional models to that of the national model requires choice of a statistical measure and it is not clear what that choice should be. Third, the best chance for improving choice of alternative constraining procedures seems to be the use of smaller sythetic models in which numerous solutions can be obtained in a Monte-Carlo fashion.

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